03/10/2020

Last Time: Exam II

Today: Sequences, Series, Chapter 11.

Future: Exam III on 04/09

Hw Due M after Spring Break

Sequences:

A list of numbers:

Ex: $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\} = \{\frac{1}{n}\}_{n=1}^{\infty}$

$\{7, \pi, e^2, 3.5, 0, 2.1, -5\}$

$\{a_n\} = \{\frac{n+2}{n^2}\}_{n=2}^{5} = \{\frac{4}{4}, \frac{5}{9}, \frac{6}{16}, \frac{7}{25}\}$

Note: A sequence has a beginning.
Functions often do not.

Find the 101^{st} term of $a_n = \left\{\frac{(-1)^n(n+1)}{\sqrt{n-1}}\right\} = \frac{-102}{10}$

" " " " " $b_n = \frac{(n+2)!}{n!} = \frac{103!}{101!} = \frac{103 \cdot 102 \cdot \cancel{101 \cdots 1}}{\cancel{101 \cdot 100 \cdots 1}}$

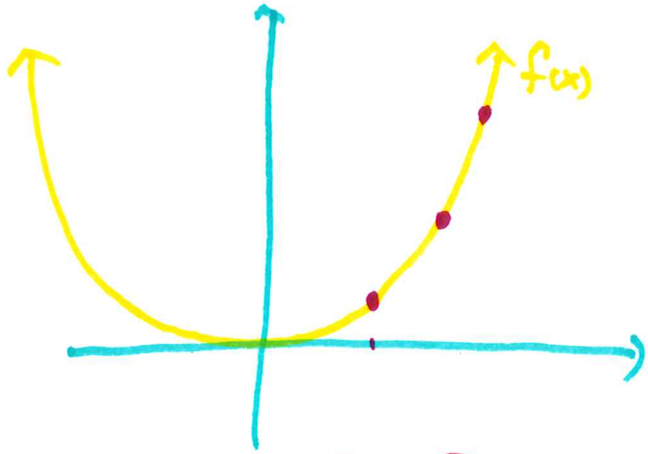
$= \boxed{103 \cdot 102}$

$0! = 1$

$\frac{(n+1)!}{n!} = n+1 \overset{n=0}{\Rightarrow} \frac{1!}{0!} = 1 \Rightarrow \boxed{1 = 0!}$

In many ways, a sequence is like a function

$a_n = n^2$, $f(x) = x^2$



$a_n = f(x)$ so long as x is a positive integer

For us, we are most interested in the limit of a sequence as $n \to \infty$; $\lim_{n\to\infty} a_n = \underbrace{\lim a_n}$

- If limit exists, we get one finite number
- If limit DNE, we get ∞, $-\infty$, oscillates.

Find the limits of:

$a_n = (-1)^n$ DNE

$c_n = (-2)^n$ DNE

$e_n = \frac{\ln(n)}{n}$

$b_n = n^2$ DNE

$d_n = \frac{1}{2^n} \to 0$

$f_n = \frac{(-1)^n}{n}$

$$\lim \frac{\ln(n)}{n} = \lim_{x\to\infty} \frac{\ln(x)}{x} \overset{L.R.}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \frac{0}{1} = \boxed{0}$$

$$\lim \frac{(-1)^n}{n} = \lim_{x\to\infty} \frac{(-1)^x}{x}$$ STOP,

$(-1)^x \to (-1)^{1/2} = i$

$(-1)^{1/4} = (i)^{1/2}$

$(-1)^{\sqrt{2}} = ?$

$(-1)^{\pi} = ?$

$(-1)^x$ is a great headache we want to avoid.

$(-1)^n$ is perfectly fine.

$x!$ is a nightmare

$n!$ is perfectly fine.

Q: How do we find $\lim \frac{(-1)^n}{n}$?

Squeeze Theorem

$$-1 \le (-1)^n \le 1$$

$$\therefore \quad \frac{-1}{n} \le \cancel{\lim} \frac{(-1)^n}{n} \le \frac{1}{n}$$

$$\therefore \quad \lim \frac{-1}{n} \le \lim \frac{(-1)^n}{n} \le \lim \frac{1}{n}$$

$$0 \le \lim \frac{(-1)^n}{n} \le 0$$

$$\therefore \lim \frac{(-1)^n}{n} = 0$$

$$\lim \frac{3n+1}{4n^2+1} \rightarrow \lim_{x\to\infty} \frac{3x+1}{4x^2+1} \overset{L.R.}{=} \lim_{x\to\infty} \frac{3}{8x} = \boxed{0}$$

$$\lim n-\sqrt{n^2-1} \rightarrow \lim_{x\to\infty} x-\sqrt{x^2-1} \cdot \left(\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right) = \lim_{x\to\infty} \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}}$$

$$= \lim_{x\to\infty} \frac{1}{x+\sqrt{x^2-1}} = \frac{1}{\infty+\infty} = \boxed{0}$$

$$\lim \frac{(-1)^n+10n}{5n+2}$$

$$= \boxed{2}$$

Squeeze Theorem:

$$-1 \le (-1)^n \le 1$$

$$\frac{-1+10n}{5n+2} \le \frac{(-1)^n+10n}{5n+2} \le \frac{1+10n}{5n+2}$$

$$2 \le \lim \frac{(-1)^n+10n}{5n+2} \le 2$$

$$\lim (-1)^n\left(\frac{2n}{5n-1}\right)$$

$$-1 \le (-1)^n \le 1$$

If n even, $\rightarrow 2/5$

If n odd, $\rightarrow -2/5$

$\therefore$ DNE

$$-\left(\frac{2n}{5n-\cancel{0}}\right) \le (-1)^n\left(\frac{2n}{5n-1}\right) \le \frac{2n}{5n-1}$$

$$-\frac{2}{5} \le \lim (-1)^n\left(\frac{2n}{5n-1}\right) \le \frac{2}{5}$$

SQUEEZE THRM FAILS

$-1 \le \sin(n) \le 1$; $0 \le \cos^2(n) \le 1$; $\lim \tan^{-1}(n) = \frac{\pi}{2}$

Find $\lim \cos\left(\sqrt{\frac{3n}{n^2+1}}\right) = \cos\left(\sqrt{0}\right) = \cos(0) = \boxed{1}$

Find $\lim \left(\frac{n+3}{n-2}\right)^n \to 1^\infty$ } I.F.

$$\lim \left(1+\frac{a}{n}\right)^{bn} = e^{ab}$$

$$\lim \left(1+\frac{1}{n}\right)^n = e$$

$$\lim \frac{\left(\frac{n+3}{n}\right)^n}{\left(\frac{n-2}{n}\right)^n} =$$

$$= \lim \frac{\left(1+\frac{3}{n}\right)^n}{\left(1+\frac{-2}{n}\right)^n} = \frac{e^3}{e^{-2}} = \boxed{e^5}$$

Now, we are interested in <u>infinite series</u>, aka adding an infinite # of #'s.

sum $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + a_4 + \cdots$

- ∞ or $-\infty$
- oscillates
- one finite number.

product $\prod$

① $\sum_{1}^{\infty} n^2 = 1 + 4 + 9 + 16 + \cdots = \infty \Leftrightarrow$ Diverges

② $\sum_{1}^{\infty} (-1)^{n-1} = 1 - 1 + 1 - 1 + 1 - 1 + \cdots$, osillates btwn 0 + 1, ∴ Diverges.

③ $\sum_{1}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots \Rightarrow$ Converges to 1

$\underbrace{\frac{1}{2} + \frac{1}{4}}_{3/4}$

$\underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8}}_{7/8}$

$\underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}}_{15/16}$

$\underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}}_{31/32}$

Sequences are <u>not</u> Series.