The following questions are to serve as a review. The material on the exam will extend beyond the scope of this review.

1. (8 points) Find the limit of the sequence \( a_n = \sqrt{16n^2 + 3n + 5} - 4n \).
   
   (A) 0  (B) \( \frac{3}{8} \)  (C) \( \frac{1}{4} \)  (D) \( \frac{2}{7} \)  (E) \( \frac{2}{5} \)  (F) \( \frac{1}{3} \)

2. (8 points) Find \( \sum_{n=2}^{\infty} \frac{1 + 3^n}{4^n} \).
   
   (A) 0  (B) \( \frac{10}{3} \)  (C) \( \frac{7}{3} \)  (D) \( \frac{9}{4} \)  (E) 3  (F) 4  (G) None of these
3. (8 points) The graph of the function \( y = \frac{2 \tan^{-1}(x)}{2x + 1} \) is shown below.

Which of the following applies to the series \( \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2 \tan^{-1}(n)}{2n + 1} \)?

(A) Absolutely Conv. (B) Conditional Conv. (C) Divergent (D) None of the above

4. (8 points) Consider the following series: \( \sum_{n=3}^{\infty} \frac{\sqrt{n^8 + n^5 + n}}{n^4 + 5n + 1} \). Which is true?

(A) The series is convergent (B) The series is divergent (C) Cannot be determined

5. (8 points) Consider the three series:

A) \( \sum_{n=3}^{\infty} \frac{5}{n \ln n} \)

B) \( \sum_{n=3}^{\infty} \frac{5}{n \ln^2 n} \)

C) \( \sum_{n=3}^{\infty} \frac{n}{\ln^3 (n)} \)

Which series converges?

(A) None (B) A only (C) B only (D) C only (E) A and B (F) A and C (G) B and C A, B, and C
6. (8 point) Use the degree 2 Taylor Polynomial of \( f(x) = x^{4/3} \) centered at \( x = 1 \) to approximate the value of \( 2^{4/3} \).

(A) \( \frac{19}{10} \)  (B) \( \frac{25}{9} \)  (C) \( \frac{8}{3} \)  (D) \( \frac{23}{9} \)  (E) \( \frac{22}{9} \)  (F) \( \frac{17}{10} \)  (G) None of These

7. Consider the series \( \sum_{n=0}^{\infty} \frac{(n+1)!}{(n^2)!}. \)

(A) The series converges  (B) The series diverges

8. Find the interval of convergence of \( \sum_{n=1}^{\infty} \frac{n}{2n(n^2 + 1)} x^n. \)

(A) \([-2, 2]\)  (B) \((-2, 2]\)  (C) \([-2, 2)\)  (D) \((-2, 2)\)

(E) \([-1/2, 1/2]\)  (F) \((-1/2, 1/2]\)  (G) \([-1/2, 1/2)\)  (H) \((-1/2, 1/2)\)
9. (8 points) Find a power series representation of $f(x) = \frac{1}{(2 - x)^2}$

\[
\begin{align*}
(A) & \sum_{n=1}^{\infty} \frac{n}{2^{n+1}} x^{n-1} \\
(B) & \sum_{n=1}^{\infty} \frac{1}{2^{2n+1}} x^{2n-2} \\
(C) & \sum_{n=1}^{\infty} \frac{(-1)^n}{2^{2n+1}} x^{2n-2} \\
(D) & \sum_{n=1}^{\infty} \frac{(-1)^n}{2^{2n+1}} x^{2n+2} \\
(E) & \sum_{n=1}^{\infty} \frac{(-1)^n n + 1}{2^{n+2}} x^n \\
(F) & \left( \sum_{n=1}^{\infty} \frac{1}{2^{n+1}} x^n \right)^2 \\
(G) & \left( \sum_{n=1}^{\infty} \frac{(-1)^n}{2^{n+1}} x^n \right)^2 \\
(H) & \sum_{n=1}^{\infty} \frac{n + 1}{2^{n+2}} x^n
\end{align*}
\]

10. (8 points) Consider the three statements:

A) The series $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$ converges by the $n^{th}$ term test.

B) The series $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$ converges by the Limit Comparison Test and geometric series test, where $b_n = \frac{1}{2^{n-1}}$.

C) The series $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$ converges by the Basic Comparison Test and geometric series test, where $b_n = \frac{1}{2^{n-1}}$.

Which statement(s) is(are) true?

(A) None  (B) A only  (C) B only  (D) C only  (E) A and B  
(F) A and C  (G) B and C  (H) A, B, and C
11. (6 points) Find the radius of convergence of \( \sum_{n=0}^{\infty} \frac{3^n}{5^{2n}} x^n \).

12. (6 points) Find the coefficient of \( x^8 \) in the Taylor Series expansion of \( \cos(x^2) \).

13. (6 points) The first degree Taylor polynomial of \( f(x) = \sqrt{x} \) centered at \( x = 4 \) is \( T_1(x) = 2 + \frac{1}{4}(x-4) \). What estimate does Taylor’s Inequality give for \( |R_2(x)| = |T_2(x) - f(x)| \) for \( 3 \leq x \leq 5 \)?

14. (6 points) Consider the sequence \( a_n = \frac{(-1)^n + 7n}{3n + 2} \). We can show it converges to \( \frac{7}{3} \) by finding sequence \( b_n \) and \( c_n \) such that \( b_n \leq a_n \leq c_n \) and using the squeeze theorems. Find \( b_n \) and \( c_n \).
Multiple Choice Answer Page.

1) __________

2) __________

3) __________

4) __________

5) __________

6) __________

7) __________

8) __________

9) __________

10) __________

Short Answers:

11) ____________________

12) ____________________

13) ____________________

14) $b_n =$ __________ $c_n =$ __________