Goals

1. Define limits geometrically
2. Compute limits algebraically
3. "Real life" applications: slope, velocity

"Def": We say \( \lim_{x \to a} f(x) = L \) if

\[ L \text{ is the expected value of } f(x) \text{ at } x = a. \]

\[ \lim_{x \to 2} f(x) = 3 \]

\[ f(x) \text{ is not defined at } x = 2, \text{ but that's } \text{OK!} \]
(a) \[ \lim_{x \to 0} \frac{|x|}{x} \quad \text{(Jump discontinuity)} \]

(b) \[ \lim_{x \to 0} \frac{1}{x} \quad \text{DNE} \]

(c) \[ \lim_{x \to 0} \sin \left( \frac{1}{x} \right) \quad \text{DNE} \]

"Def. (A):" \[ \lim_{x \to a^-} f(x) = L_1 \quad \text{if the expected value from the left is } L_1 \]

\[ \lim_{x \to a^+} f(x) = L_2 \quad \text{if the expected value from the right is } L_2 \]
\[ \lim_{x \to 2^-} f(x) = -1 \]
\[ \lim_{x \to 2^+} f(x) = 8 \]
\[ \lim_{x \to 10^-} f(x) = \frac{1}{2} \]
\[ \lim_{x \to 10^+} f(x) = \frac{1}{2} \]

\[ f(2) = 1 \]
\[ f(10) = 1 \]

**Example:**
\[ \lim_{x \to -1} \frac{3x - 1}{x + 2} = \frac{(3)(-1) - 1}{(-1) + 2} = \frac{-4}{1} = -4 \]

**Step 1:** If function is given by a single rule, then plug in "a".

**Step 2:** If you get a number, you are done.
If you don't get a number (e.g. undefined), then haha, pray, simplify...
slope of a line passing thru \((x_1, y_1)\) and \((x_2, y_2)\)

\[\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}\]

slope of tangent:

\[\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} x + 1 = 2\]