Find Absolute min, max of \( f(x) = x^3 - 3x + 2 \) on \([0, 3] \).

\[
f'(x) = 3x^2 - 3
\]

\[
\begin{align*}
3x^2 - 3 &= 0 \\
3(x^2 - 1) &= 0 \\
3(x+1)(x-1) &= 0
\end{align*}
\]

\[x = -1, 1\]

Once we have critical numbers (x-values), we need critical values (y-values). We will plug x-values into \( f(x) \) NOT \( f'(x) \).

I only care about the interval \([0, 3] \).

Critical number \( x = 1 \Rightarrow f(1) = 0 \)

Endpoints \( x = 0 \Rightarrow f(0) = 2 \)

\( x = 3 \Rightarrow f(3) = 20 \)

Abs min of \( 0 \) at \( x = 1 \)
Abs max of \( 20 \) at \( x = 3 \)

\[
\begin{array}{c}
0 \\
\uparrow \\
\downarrow
\end{array}
\]

\[
\begin{array}{c}
3 \\
\downarrow \\
20
\end{array}
\]
Rolle's Theorem: If \( f(x) \) is a function such that:

1. It is continuous on \([a,b]\)
2. It is differentiable on \((a,b)\)
3. \( f(a) = f(b) \)

Then there is an \( x \)-value \( c \) such that \( a < c < b \) and \( f'(c) = 0 \).

Mean Value Theorem: If \( f(x) \) is a function such that:

1. It is continuous on \([a,b]\)
2. It is differentiable on \((a,b)\)

Then there is an \( x \)-value \( c \) such that \( a < c < b \) and

\[
\frac{f(b) - f(a)}{b - a} = \frac{\Delta y}{\Delta x} = f'(c)
\]

\( \iff f(b) - f(a) = f'(c)(b-a) \)
Let $f(x) = x^3 - x$ on $[0, 2]$. Find the value of $c$ which satisfies the Mean Value Theorem, MVT.

\[ f'(c) = \frac{f(b) - f(a)}{b - a} \]

\[ \frac{f(2) - f(0)}{2 - 0} = \frac{6 - 0}{2 - 0} = 3 \]

\[ f'(c) = 3, \quad f'(x) = 3x^2 - 1 \]

\[ 3 = 3x^2 - 1 \]

\[ 4 = 3x^2 \]

\[ x^2 = \frac{4}{3} \quad \Rightarrow \quad x = \pm \sqrt{\frac{4}{3}} = \pm \frac{2}{\sqrt{3}} \]

\[ c = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \]
A silly, but true, consequence of the M.V.T.

**Theorem:** If \( f'(x) = 0 \) for all \( a < x < b \), then \( f(x) \) is the constant function.

**Why:** By MVT, \( f'(c) = \frac{f(b) - f(a)}{b - a} \) \( \iff \)

\[ f(b) - f(a) = f'(c)(b - a) \]

\[ f(b) - f(a) = 0 \]

\[ f(b) = f(a) \] Always, for all \( a, b \).

What can you conclude about \( f(x) = \frac{x + 3}{x - 2} \) [L.4]

from the M.V.T.

\[ 1 - x - x^3 \text{ on } [-2, 1] \]

\[ f'(c) = \frac{f(1) - f(-2)}{1 - (-2)} = \frac{-1 - [1 - (2) - (-2)^3]}{3} = \frac{-1 - [1 + 2 + 8]}{3} = \frac{-1 - 11}{3} = -4 \]

\[ -1 - 3x^2 = -4 \]

\[ -3x^2 = -3 \]

\[ x^2 = 1 \]

\[ x = 1, -1 \]

\[ c = -1 \]
what does the derivative say about the graph?

Given an interval \([4, 5]\)

1. If \(f'(x) > 0\) for all \(a < x < b\), then \(f(x)\) is increasing.
2. If \(f'(x) < 0\), then \(f(x)\) is decreasing.

Determine the intervals where \(f(x) = 3x^4 - 4x^3 - 12x^2 + 5\)

is increasing and where it is decreasing.

\[
f'(x) = 12x^3 - 12x^2 - 24x = 0
\]

\[
= 12(x^3 - x^2 - 2x) = 0
\]

\[
= 12x(x^2 - x - 2) = 0
\]

\[
= 12x(x + 1)(x - 2) = 0
\]

\[x = 0, -1, 2 \quad \text{critical numbers}\]

**Number Line Technique:**

\[\begin{array}{cccccccccc}
& -2 & -1 & \frac{1}{2} & 0 & 1 & 2 & \text{sup} & \text{sup} & \\
\text{local min} & \text{local max} & & & & & & & & \\
\end{array}\]

\[
f'(0) = 12(0^3) - 12(0^2) - 24(0) = -24
\]

\[
f'(\frac{1}{2}) = 12(\frac{1}{8}) (\frac{1}{2} - 1)(\frac{1}{2} - 2)
\]

\[
= \frac{12}{8} \left( -\frac{1}{2} \right) \left( -\frac{3}{2} \right) = -3
\]

\[
f'(-2) = 12(-2)(-2 + 1)(-2 - 2)
\]

\[
= [+] [-] [-] = -
\]

\[-\infty < x < -2 \rightarrow \text{dec} \]

\[-2 < x < 0 \rightarrow \text{Inc} \]

\[0 < x < 2 \rightarrow \text{Dec} \]

\[2 < x < \infty \rightarrow \text{Inc} \]