

M341 Second **Midterm Exam**, April 7, 2005

1. Calculate the volume of the parallelepiped determined by the vectors

$$x = [2, 7, -1], \quad y = [0, 2, -3], \quad z = [0, 2, 4].$$

2. If A is a 4×4 matrix with determinant 4, what is $|3A|$? Why?

3. Calculate the determinant of $A = \begin{bmatrix} 5 & -6 & -3 \\ 4 & 6 & 1 \\ 5 & 10 & 2 \end{bmatrix}$ by row reducing A to upper triangular form.

4. Prove that if A and B are $n \times n$ matrices, then $|A B^T| = |A^T B|$.

5. Find the inverse of $A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 2 & 2 \\ -3 & 0 & 8 \end{bmatrix}$ by first computing the adjoint matrix \mathcal{A} for A .

6. Prove that an $n \times n$ matrix A is singular if and only if $\lambda = 0$ is an eigenvalue for A .

7. Use cofactor expansion along any row or column to find the determinant of

$$A = \begin{bmatrix} 3 & 0 & -4 & 6 \\ 4 & -2 & 5 & 2 \\ -3 & 3 & 0 & -5 \\ 7 & 0 & 0 & -8 \end{bmatrix}.$$

Be sure to use cofactor expansion to find any 3×3 determinants needed *as well*.

8. Let $A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 2 & 2 \\ -3 & 0 & 8 \end{bmatrix}$. Find a nonsingular matrix P having all integer entries, and a diagonal matrix D such that $D = P^{-1}AP$.

9. Prove that the set of $n \times n$ symmetric matrices is a subspace of \mathcal{M}_{nn} .

10. Find a simplified general form for all the vectors in $\text{span}(S)$ in \mathcal{P}_3 if

$$S = \{4x^3 - 20x^2 - x - 11, 6x^3 - 30x^2 - 2x - 18, -5x^3 + 25x^2 + 2x + 16\}.$$