METHODS OF APPLIED MATHEMATICS II

MATH 383D, Unique #55275 and CAM 385D, Unique #61120
Spring 2001

Instructors:
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Office Hours: Open office hours: unless the instructors have pressing business, they will be available to help students who come by. Students may also make specific appointments for help.

Meeting: TTh 9:30–11:00, RLM 10.176, and a weekly problem discussion session will be scheduled.

Textbook: Lecturer-prepared notes.

Homework, Exams, and Grades: Homework will be assigned regularly. Students are encouraged to work in groups; however, each student must write up his or her own work. One mid-term exam will be given. The final exam will be given during finals week (officially scheduled for Friday, May 11, 9:00–12:00 noon). The final grade will be based on the homework and the two exams.

Course Description: This is the second semester of a course on methods of applied mathematics. It is open to mathematics, science, engineering, and finance students. It is suitable to prepare graduate students for the Applied Mathematics Preliminary Exam in mathematics and the Area A Preliminary Exam in CAM.

1. *The Fourier Transform and Sobolev Spaces* (4 weeks)
   1. The Schwartz space and tempered distributions.
   2. The Fourier transform.
   3. The Plancherel Theorem.
   4. Convolutions.
   5. Fundamental solutions of PDE's.
   6. Sobolev spaces.
   7. Imbedding Theorems.
   8. The Trace Theorem.
2. *Variational Boundary Value Problems (BVP)* (3 weeks)
   1. Weak solutions to elliptic BVP's.
   2. Variational forms.
   3. Lax-Milgram Theorem.
   4. Galerkin approximations.
   5. Green's functions.
3. *Differential Calculus in Banach Spaces and Calculus of Variations* (4 weeks)
   1. The Fréchet derivatives.
   2. The Chain Rule and Mean Value Theorems.
   3. Higher order derivatives and Taylor's Theorem.
   4. Banach's Contraction Mapping Theorem and Newton's Method.
   5. Inverse and Implicit Function Theorems, and applications to nonlinear functional equations.
   6. Extremum problems, Lagrange multipliers, and problems with constraints.
   7. The Euler-Lagrange equation.
   8. Applications to classical mechanics and geometry.
4. *Asymptotic Analysis* (3 weeks)
   1. Definitions and fundamental properties.
   2. Examples of transcendental equations and initial-value problems.
   3. Boundary layers in regular and singular perturbations.

4. Perturbation methods for linear eigenvalue problems.

References:

1. R. A. Adams, *Sobolev Spaces*, Academic Press, 1975.
2. J.-P. Aubin, Applied Functional Analysis, Wiley, 1979.
3. C. Carathéodory, *Calculus of Variations and Partial Differential Equations of the First Order*, 1982.

*4. L. Debnath and P. Mikusiński, *Introduction to Hilbert Spaces with Applications*, Academic Press, 1990.

5. G.B. Folland, *Introduction to Partial Differential Equations*, Princeton, 1976.
6. I.M. Gelfand and S.V. Fomin, *Calculus of Variations*, Prentice-Hall, 1963.
7. J. Jost and X. Li-Jost, Calculus of Variations, Cambridge, 1998,
8. E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley, 1978.

*9. E.H. Lieb and M. Loss, *Analysis*, AMS, 1997.

*10. J.T. Oden & L.F. Demkowicz, *Applied Functional Analysis*, CRC Press, 1996.

*11. F.W.J. Olver, *Asymptotics and Special Functions*, Academic Press, 1974.

*12. M. Reed & B. Simon, *Methods of Modern Physics, Vol.* 1*, Functional analysis.*

*13. W. Rudin, *Functional Analysis*, McGraw Hill, 1991.

*14. W. Rudin, *Real and Complex Analysis*, 3*rd* Ed., McGraw Hill, 1987.

15. H. Sagan, Introduction to the Calculus of Variations, Dover, 1969.
16. R.E. Showalter, *Hilbert Space Methods for Partial Differential Equations*, available at World Wide Web address `http://ejde.math.swt.edu//mono-toc.html`.
17. E. Stein and G. Weiss, *Introduction to Fourier Analysis on Euclidean Spaces*, Princeton, 1971.
18. K. Yosida, *Functional Analysis*, Springer-Verlag, 1980.

- See also: E.W. Cheney and H.A. Koch, *Notes on Applied Mathematics*, Department of Mathematics, University of Texas at Austin.
  (*) On reserve in PMA Library