

Trigonometric Identities

You should know and be able to use each of the following trig identities.

Odd-even identities:

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

Co-function identities:

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

Pythagorean identities:

$$\sin^2 x + \cos^2 x = 1$$

From this follow

$$\sec^2 x = 1 + \tan^2 x$$

$$\csc^2 x = 1 + \cot^2 x$$

Addition formulas:

$$\sin(A + B) = \sin A \cos B + \sin B \cos A$$

$$\sin(A - B) = \sin A \cos B - \sin B \cos A$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Many useful identities follow from the addition formulas:

Double-angle formulas: setting $A = B$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

Alternative very useful versions of the last two are

$$\cos^2 x = \frac{1}{2}(\cos 2x + 1)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

Product formulas:

$$\sin x \cos y = \frac{1}{2}(\sin(x + y) + \sin(x - y))$$

$$\cos x \sin y = \frac{1}{2}(\sin(x + y) - \sin(x - y))$$

$$\cos x \cos y = \frac{1}{2}(\cos(x + y) + \cos(x - y))$$

$$\sin x \sin y = -\frac{1}{2}(\cos(x + y) - \cos(x - y))$$