# Fun with Congruences

1. Reduce 21761 (mod 7)

2. Reduce $8^{133} \pmod{10}$

3. Reduce $28! \pmod{29}$

4. Reduce $\sum_{i=1}^{100} i^2 \pmod{17}$

5. Find a complete residue system modulo 7 consisting of primes.

6. Prove that for an odd prime $p$, $1 + 2 + 3 + \cdots + (p-1) \equiv 0 \pmod{p}$.

7. For an odd prime $p$, prove that $(a+b)^p \equiv a^p + b^p \pmod{p}$.

8. Find the remainder when $3^{37}$ is divided by 26 and when $2^{43}$ is divided by 11.

9. Use Euler's Theorem to solve the congruences $5x \equiv 7 \pmod{12}$, $7x \equiv 1 \pmod{10}$, and $8x \equiv 4 \pmod{5}$.

10. Find all incongruent solutions to $x^5 \equiv 1 \pmod{11}$ and $x^5 \equiv 1 \pmod{121}$.

11. Find all incongruent solutions to $12x \equiv 98 \pmod{20}$.

12. Solve the system:

$$\begin{aligned} x &\equiv 28 \pmod{29} \\ x &\equiv 29 \pmod{30} \\ x &\equiv 10 \pmod{11} \end{aligned}$$

13. Solve the system:

$$\begin{aligned} 5x &\equiv 11 \pmod{17} \\ 3x &\equiv 19 \pmod{32} \\ 11x &\equiv 6 \pmod{37} \end{aligned}$$

14. Find the smallest positive integer that has remainders 5, 4, 3, and 2 when divided by 6, 5, 4, and 3, respectively.

15. Find all the primes which are the difference of the fourth powers of two integers.

16. Reduce $2^{987654321} \pmod{9}$

17. Reduce $6^{83} + 8^{83} \pmod{49}$

18. Which of the numbers in the list 121, 1221, 12221, 122221, ... are perfect squares?

20. If $p$ and $q$ are distinct primes, prove that $p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$.

21. Use Fermat's method to factor 6077, $3,200,399$ and $24,681,023$.