

M328K Sample Test 3

No Calculators, books, notes, etc.

1. a. Compute $\phi(7)$, $\phi(18)$ and $\phi(2^{10})$.

b. Compute $\sigma(7)$, $\sigma(18)$ and $\sigma(2^{10})$.

c. Compute $\tau(7)$, $\tau(18)$ and $\tau(2^{10})$.

2. Find $\overline{13}$ modulo 100.

3. State and prove Wilson's Theorem.

4. Use the Pollard $p - 1$ method to find a factor of 323.

5. Prove that if p is prime, then $1^{p-1} + 2^{p-1} + \dots + (p-1)^{p-1} \equiv -1 \pmod{p}$.

6. Use the Chinese Remainder Theorem and Hensel's Lemma to find all solutions to $x^2 + 4x + 10 \equiv 0 \pmod{75}$.

7. Find all solutions to $\phi(n) = 3$.

8. A *repunit* is a positive integer of the form $111\dots 1$.

a. Find, with proof, all prime repunits with an even number of digits.

b. Find, with proof, all prime repunits whose number of digits is divisible by 3.

c. Prove or disprove: If a repunit is prime, it has a prime number of digits.