

M328K Sample Final

Take with grain of salt.

No Calculators, books, notes, etc.

1. Write the Cantor expansion of 303.
2. Evaluate $\phi(\phi(13))$.
3. Evaluate $\sigma(490)$.
4. Write the prime factorization of $2^{36} - 1$.
5. Write a linear Diophantine equation in two variables which has no solution.
6. Find the least positive residue of $3^{654317398}$ modulo 11.
7. If $7^\alpha \parallel 1000!$, find α .
8. Write the prime factorization of 1501500.
9. Write the 20th Fibonacci number.
10. Find a reduced residue system modulo 18.
11. Express $(24, 92)$ as linear combination of the two numbers.
12. Find the least nonnegative solution to the system:
$$\begin{aligned}x &\equiv 1 \pmod{5} \\x &\equiv 2 \pmod{6} \\x &\equiv 3 \pmod{75}\end{aligned}$$
13. Find all solutions to $5x \equiv 7 \pmod{11}$.
14. Find all solutions to $x^4 - 2x^3 + 2x^2 + 3x + 2 \equiv 0 \pmod{3}$ and $\pmod{9}$.
15. Factor 7535009. (It's square root is 2744.997...)
16. Prove: If c and d are integers and $c = dq + r$, then $(c, d) = (d, r)$.
17. Prove: Prove that congruence modulo m is an equivalence relation.
18. Prove: If n and k are positive integers with $n \geq k$, then $\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$.
19. Prove: If p is prime and $(a, p) = 1$, then $1 + a + a^2 + \cdots + a^{p-2} \equiv 0 \pmod{p}$.
20. Find a primitive root of 23.
21. Find all the quadratic residues of 23.