

M427L Sample Test

Note: Bart doesn't answer questions about the Sample Test.

1. Let C be a curve connecting the points $(0, 2, 1)$ and $(1, \pi/2, 2)$. Let $\mathbf{F} = 2 \cos y \mathbf{i} + (1/y - 2x \sin y) \mathbf{j} + (1/z) \mathbf{k}$. Find the work done by \mathbf{F} moving a particle along C .
2. Let W be the solid between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$ and above $z = \sqrt{x^2 + y^2}$. If the solid has density given by $\delta(x, y, z) = 1/(x^2 + y^2 + z^2)$, find the mass of W .
3. Use the substitution $u = x + y$, $v = y - 2x$ to evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y}(y-2x)^2 dy dx$.
4. Let $\mathbf{F} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a C^2 vector field. Prove that $\operatorname{div} \operatorname{curl} \mathbf{F} = \mathbf{0}$.
5. *Ultrabonus* If a circle C of radius 1 rolls along the outside of the circle $x^2 + y^2 = 16$, a fixed point P on C traces out a curve called an *epicycloid*. Sketch this curve and find parametric equations for it. Then find its arclength.
6. Tell whether $\mathbf{F}(x, y, z) = e^x \ln y \mathbf{i} + \left(\sin z + \frac{e^x}{y} \right) \mathbf{j} + y \cos z \mathbf{k}$ is conservative. If so, find a potential function for \mathbf{F} .
7. A wire is shaped in a semicircle of radius 3 centered at the origin, above the x -axis. The density of the wire is given by $\delta = 5y$. Find the mass of the wire.
8. Find the work done by $\mathbf{F}(x, y) = (e^x - y^3) \mathbf{i} + (\cos y + x^3) \mathbf{j}$ on a particle that travels once counterclockwise about the circle $x^2 + y^2 = 1$.
9. Rewrite $\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} f(x, y, z) dz dy dx$ with order of integration $dx dy dz$.
10. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2 + y^2 + z^2)^2 dz dy dx$.
11. Tell whether $\mathbf{F}(x, y, z) = (2xz + \sin y) \mathbf{i} + x \cos y \mathbf{j} + x^2 \mathbf{k}$ is conservative. If so, find a potential function for \mathbf{F} .
12. Evaluate $\int_0^1 \int_{2y}^2 4 \cos(x^2) dx dy$.
13. Find the curvature of $\mathbf{c}(t) = \langle t, t^2, 1 - t^2 \rangle$ at the point where $t = 1$.
14. Let C be the curve given by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$, $0 \leq t \leq 1$, and let $\mathbf{F}(x, y, z) = xyz \mathbf{i} + yz \mathbf{j} + zx \mathbf{k}$. Evaluate $\int_C \mathbf{F} \bullet ds$.
15. Evaluate $\int_C x^2 dx + yz dy + (y^2/2) dz$ along the line segment C joining $(0, 0, 0)$ and $(0, 3, 4)$.
16. Let C be the curve of the intersection of $(x - 1)^2 + 4y^2 = 16$ and $2x + y + z = 3$, oriented counter-clockwise when viewed from above. Let $\mathbf{F}(x, y, z) = (z^2 + y^2 + \sin x^2) \mathbf{i} + (2xy + z) \mathbf{j} + (xz + 2yz) \mathbf{k}$. Evaluate $\oint_C \mathbf{F} \bullet ds$.
17. Find a parametrization of the surface $x^3 + 3xy + z^2 = 2$.
18. Find parametric equations of the curve which is the intersection of the surfaces $x^2 + y^2 = 1$ and $x + y + z = 1$.
19. Use a quadratic approximation for $f(x, y, z) = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ to estimate $\frac{1}{0.9} + \frac{1}{1.05} + \frac{1}{2.2}$.
20. Find and classify the critical points of $f(x, y, z) = x^2 y^2 - z^4 - 16x + 16y + 2z^2$.