1. (5 points) If $y_0$ is the solution of the equation

$$x y' + 2y = 5x \quad y(1) = 8,$$

determine the value of $y_0(2)$.

2. (5 points) Find the area enclosed by the curve

$$x = 1 + \sin(t), \quad y = \sin(2t), \quad \text{where} \quad 0 \leq t \leq \pi.$$
M408D - Quiz 5 Solutions - 15 minutes

Grading scheme for question 1: Calculate the integrating factor right - 2 points, find the formular for \( y \) - 2points, compute the value right - 1point.

Reference: The first questions from this Quiz are from Quest.
This print-out should have 2 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

**BusC7c11**

001 10.0 points

If $y_0$ is the solution of the equations

$$xy' + 2y = 5x, \quad y(1) = 8,$$

determine the value of $y_0(2)$.

1. $y_0(2) = \frac{19}{4}$

2. $y_0(2) = \frac{29}{6}$

3. $y_0(2) = \frac{59}{12}$ correct

4. $y_0(2) = \frac{14}{3}$

5. $y_0(2) = \frac{55}{12}$

**Explanation:**

After dividing we see that the given differential equation becomes

$$(\dagger) \quad y' + \frac{2}{x}y = 5,$$

which is a first order linear equation having integrating factor

$$\mu(x) = \exp\left(\int \frac{2}{x} \, dx\right) = x^2.$$

With this $(\dagger)$ becomes

$$x^2y' + 2xy = 5x^2.$$

Thus

$$x^2y = \int 5x^2 \, dx = \frac{5x^3}{3} + C,$$

where $C$ is an arbitrary constant. For $y_0$ the value of $C$ is determined by the condition $y(1) = 8$. Consequently,

$$y_0(x) = \frac{1}{3}\left(5x + \frac{8(3) - 5}{x^2}\right).$$

At $x = 2$, therefore,

$$y_0(2) = \frac{59}{12}.$$

**BusC7c09**

002 10.0 points

If $y_1$ is the particular solution of the differential equation

$$\frac{dy}{dx} - \frac{2y}{x} = 4x^2 - 2$$

which satisfies $y(1) = 4$, determine the value of $y_1(2)$.

1. $y_1(2) = 24$

2. $y_1(2) = 26$

3. $y_1(2) = 25$

4. $y_1(2) = 28$ correct

5. $y_1(2) = 27$

**Explanation:**

The integrating factor needed for the first order differential equation $(\ddagger)$ is

$$\mu(x) = e^{-\int \frac{2}{x} \, dx} = e^{-2\ln x} = \frac{1}{x^2}.$$

After multiplying both sides of $(\ddagger)$ by $1/x^2$ we can thus rewrite the equation as

$$\frac{d}{dx}\left(\frac{y}{x^2}\right) = 4 - \frac{2}{x^2}.$$

Consequently, the general solution of $(\ddagger)$ is given by

$$\frac{y}{x^2} = 4x + \frac{2}{x} + C$$

where $C$ is an arbitrary constant. For the particular solution $y_1$ the value of $C$ is determined by the condition $y(1) = 4$ since

$$y(1) = 4 \quad \Rightarrow \quad 4 = 6 + C,$$

and so

$$y_1(x) = x^2\left(4x + \frac{2}{x} - 2\right).$$

At $x = 2$, therefore,

$$y_1(2) = 28.$$
Figure 1: Left: parametric curve: \( x = 1 + \sin t \), \( y = \sin(2t) \), where \( 0 \leq t \leq \pi \). Right: parametric curve \( x = \frac{1}{2}t^2 \), \( y = \frac{1}{3}t^3 \), where \(-1 \leq t \leq 1\).

**Quiz 5- Section 52790 - Question 2:** Find the area enclosed by the curve

\[ x = 1 + \sin(t), \quad y = \sin(2t), \quad \text{where} \quad 0 \leq t \leq \pi. \]

The area is

\[
\text{Area} = 2 \int_{0}^{\pi/2} \sin(2t) \cos(t) \, dt = 4 \int_{0}^{\pi/2} \sin(t) \cos^2(t) \, dt,
\]

2 (points)

Use u-sub with \( u = \cos t \). Then \( du = \sin t \, dt \).

When \( t = 0 \), \( u = \cos 0 = 1 \); when \( t = \pi/2 \), \( u = \cos(\pi/2) = 0 \).

The area is

\[
\text{Area} = -4 \int_{1}^{0} u^2 \, du = 4 \int_{0}^{1} u^2 \, du = 4 \frac{u^3}{3}\bigg|_{0}^{1} = \frac{4}{3}.
\]

2 (points)

**Quiz 5- Section 52795 - Question 2:** Find the exact length of the curve

\[ x = \frac{1}{2}t^2, \quad y = \frac{1}{3}t^3, \quad \text{where} \quad -1 \leq t \leq 1. \]

The length of the curve is

\[
\text{Length} = 2 \int_{0}^{1} \sqrt{t^2 + t^4} \, dt = 2 \int_{0}^{1} t \sqrt{1 + t^2} \, dt
\]

2 (points)

Use u-sub with \( u = t^2 + 1 \). Then \( du = 2t \, dt \).

When \( t = 0 \), \( u = 1 \); when \( t = 1 \), \( u = 2 \).

The length is

\[
\text{Length of the curve} = \int_{1}^{2} \sqrt{u} \, du = \frac{2}{3} u^{3/2}\bigg|_{1}^{2} = \frac{2}{3} \left(2\sqrt{2} - 1\right).
\]

2 (points)