This print-out should have 2 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.
Determine whether the sequence \( \{a_n\} \) converges or diverges when 

\[ a_n = (-1)^n \left( \frac{4n}{8n + 7} \right), \]

and if it does, find its limit.

**Explanation:**

After division,

\[
\frac{4n}{8n + 7} = \frac{4}{8 + \frac{7}{n}}.
\]

Now \( \frac{7}{n} \to 0 \) as \( n \to \infty \), so

\[
\lim_{n \to \infty} \frac{4n}{8n + 7} = \frac{1}{2} \neq 0.
\]

Thus as \( n \to \infty \), the values of \( a_n \) oscillate between values ever closer to \( \pm \frac{1}{2} \). Consequently, the sequence diverges.
Find a polar representation for the curve whose Cartesian equation is

\[ x^2 + (y + 1)^2 = 1. \]

**Explanation:**

We have to substitute for \( x, y \) in

\[ x^2 + (y + 1)^2 = 1 \]

using the relations

\[ x = r \cos \theta, \quad y = r \sin \theta. \]

But after expansion the Cartesian equation becomes

\[ x^2 + y^2 + 2y + 1 = 1. \]

Now \( x^2 + y^2 = r^2 \), so

\[ r^2 + 2r \sin \theta = 0, \]

which after cancellation gives the polar representation

\[ r + 2 \sin \theta = 0. \]
Determine if the sequence \( \{a_n\} \) converges, and if it does, find its limit when
\[
a_n = \frac{4n + (-1)^n}{5n + 4}.
\]

**Explanation:**
After division by \( n \) we see that
\[
a_n = \frac{4 + \frac{(-1)^n}{n}}{\frac{5}{n} + 4}.
\]

But
\[
\frac{(-1)^n}{n}, \quad \frac{4}{n} \to 0
\]
as \( n \to \infty \), so \( a_n \to \frac{4}{5} \) as \( n \to \infty \). Consequently, the sequence converges and has
\[
\text{limit} = \frac{4}{5}.
\]
Find the slope of the tangent line to the graph of \( r = \sin 2\theta \) at \( \theta = \pi/3 \).

**Explanation:**

The graph of a polar curve \( r = f(\theta) \) can expressed by the parametric equations

\[
x = f(\theta) \cos \theta, \quad y = f(\theta) \sin \theta.
\]

In this form the slope of the tangent line to the curve is given by

\[
\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta}.
\]

When \( r = \sin 2\theta \), therefore,

\[
\frac{dy}{dx} = \frac{2 \cos 2\theta \sin \theta + \sin 2\theta \cos \theta}{2 \cos 2\theta \cos \theta - \sin 2\theta \sin \theta}.
\]

Thus, at \( \theta = \pi/3 \),

\[
\frac{dy}{dx} = \frac{(-2\sqrt{3} + \sqrt{3})/4}{(-2 - 3)/4}.
\]

Consequently, at \( \theta = \pi/3 \),

\[
\text{slope} = \frac{dy}{dx}_{\theta=\pi/3} = \frac{1}{5} \sqrt{3}.
\]