Question: Find the radius of convergence, and the interval of convergence for the power series
\[ \sum_{n=2015}^{\infty} \frac{(4x + 3)^n}{n^{3/2}} \]

Answer: Apply the ratio test:

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{(4x + 3)^{n+1}}{(n+1)^{3/2}} \cdot \frac{n^{3/2}}{(4x + 3)^n} \right| = \lim_{n \to \infty} \left| \frac{(4x + 3)}{(n+1)^{3/2}} \right| = \left| \frac{4x + 3}{1} \right|
\]

So if
\[ |4x + 3| < 1 \iff -1 < 4x + 3 < 1 \iff -\frac{1}{4} < x < \frac{3}{4} \iff -1 < x < -\frac{1}{2}, \]
the power series is convergent. The radius of convergence is \( R = \frac{1}{4} \).

When \( x = -1 \), the power series becomes
\[ \sum_{n=2015}^{\infty} \frac{(-1)^n}{n^{3/2}}. \]

This is an alternating series \( \sum (-1)^n b_n \), with \( b_n = \frac{1}{n^{3/2}} \). It’s clear that
\[ b_{n+1} \leq b_n, \quad \text{and} \quad \lim_{n \to \infty} b_n = 0. \]

Apply the alternating series test, the series \( \sum_{n=2015}^{\infty} \frac{(-1)^n}{n^{3/2}} \) is convergent.

When \( x = -\frac{1}{2} \), the power series becomes
\[ \sum_{n=2015}^{\infty} \frac{1}{n^{3/2}}. \]

This is the \( p \)-series, with \( p = 3/2 \), therefore the series is convergent.

Therefore, the interval of convergence for that power series is \([-1, -\frac{1}{2}]\).

M408D - Quiz 8 - Section 52795 (1:00 pm - 2:00 pm) - 10 minutes

Question: Find the radius of convergence, and the interval of convergence for the power series (hint: \( \sqrt{n} > \ln n \) for \( n > 0 \))
\[ \sum_{n=10}^{\infty} \frac{(2x + 1)^n}{(\ln n)^2} \]

Answer: Apply the ratio test:

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{(2x + 1)^{n+1}}{(\ln(n+1))^2} \cdot \frac{(\ln n)^2}{(2x + 1)^n} \right| = \lim_{n \to \infty} \left| (2x + 1) \left( \frac{\ln n}{\ln(n+1)} \right)^2 \right|
\]
Apply the L’Hospital rule, we have
\[
\lim_{n \to \infty} \frac{\ln n}{\ln(n+1)} = \lim_{n \to \infty} \frac{\frac{1}{n+1}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{n + 1}{n} = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right) = 1.
\]
Therefore
\[
\lim_{n \to \infty} \left|\frac{a_{n+1}}{a_n}\right| = |2x + 1|.
\]
So when
\[|2x + 1| < 1 \iff -1 < 2x + 1 < 1 \iff -\frac{1}{2} < x + \frac{1}{2} < \frac{1}{2} \iff -1 < x < 0,
\]
the power series is convergent and the radius of convergent is \(\frac{1}{2}\).

At \(x = -1\), the power series becomes
\[
\sum_{n=10}^{\infty} \frac{(-1)^n}{(n \ln n)^2}
\]
This is an alternating series \(\sum (-1)^n b_n\), with \(b_n = \frac{1}{(n \ln n)^2}\). It’s clear that
\[b_{n+1} \leq b_n, \quad \text{and} \quad \lim_{n \to \infty} b_n = 0.
\]
Apply the alternating series test, the series \(\sum_{n=10}^{\infty} \frac{(-1)^n}{(n \ln n)^2}\) is convergent.

At \(x = 0\), the power series becomes
\[
\sum_{n=10}^{\infty} \frac{1}{(n \ln n)^2}
\]
Use the hint \(\ln n < \sqrt{n}\) we have
\[\ln^2 n < n \Rightarrow \frac{1}{(\ln n)^2} > \frac{1}{n}\]
The harmonic series \(\sum \frac{1}{n}\) is divergent so the series \(\sum_{n=10}^{\infty} \frac{1}{(n \ln n)^2}\) is divergent.

Therefore, the interval of convergence for that power series is \([-1, 0)\).