Research Statement
Jeffrey Danciger

My research program is dedicated to the geometry, topology, and deformation theory of *locally homogeneous geometric structures on manifolds*, a subject with roots in Felix Klein's 1872 Erlangen program that features a blend of differential geometry, Lie theory, representation theory, and dynamics. I study an array of low-dimensional geometric structures modeled on non-Riemannian geometries including semi-Riemannian, affine, and projective geometries. Most of my work is aimed at understanding and exploiting a phenomenon, known as *geometric transition*, by which different moduli spaces of geometric manifolds interact with one another.

One main branch of my recent work, joint with François Guéritaud and Fanny Kassel, studies Margulis spacetimes as rescaled limits of complete anti de Sitter (AdS) spacetimes. Margulis spacetimes are certain flat Lorentzian three-manifolds that play an important role in the study of affine geometry and the Auslander Conjecture [Au, Mil, Ma1, Ma2, Ab, GLM, CDG, AMS]. Complete AdS spacetimes, which have constant negative curvature, were studied by Kulkarni–Raymond [KR] in the context of Thurston's geometrization program and currently serve as important testing grounds for the study of co-rank one geometric structures [GK, Zeg, BBD+]. Our methods develop a fundamental link between the theory of Margulis spacetimes and of complete AdS spacetimes, leading to parallel results about the topology, geometry, and deformation theory of both classes of manifolds. Upon finishing one final case (in preparation), our work [DGK1, DGK3] will prove the *Tameness Conjecture* [DrG1] for Margulis spacetimes[1] as well as similar statements about the topology of complete AdS spacetimes. In [DGK2] (see writing sample), we give a parameterization of the moduli of Margulis spacetimes in terms of combinatorial data on an associated hyperbolic surface. As a corollary, we complete the program initiated by Drumm and Goldman to construct any Margulis spacetime using special polyhedral fundamental domains, and we also introduce AdS analogues of these polyhedral domains (see [DGK4]). In order to make progress on the basic open questions of affine geometry in higher dimensions (e.g. the Auslander, Markus, and Tameness Conjectures), we have begun work to generalize our methods to understand higher dimensional analogues of Margulis spacetimes as limits of other geometric structures.

Another central branch of my research program, rooted in my Ph.D. thesis, explores the surprising relationship between hyperbolic manifolds and incomplete anti-de Sitter space-times in dimension three. Pioneering work of Mess [Mes, ABB+, BB] demonstrated a remarkable similarity in the behavior of quasifuchsian hyperbolic manifolds [Ber] and a certain class of incomplete AdS spacetimes studied by Witten [Wit] as a simple model for quantum gravity. Stemming from Mess's work, results and questions (see e.g. [BBD+]) in hyperbolic and AdS geometry have begun to appear in tandem, suggesting the existence of a deeper link. My thesis and subsequent papers [Da1, Da2] (see writing sample) introduced a natural framework that produces continuous families of geometric structures transitioning from hyperbolic to AdS geometry. Recently, my joint work with Sara Maloni and Jean-Marc Schlenker [DMS] uses these tools (and many others) to characterize the geometry of convex ideal polyhedra in three-dimensional AdS geometry following Rivin's celebrated results [Riv] on ideal polyhedra in hyperbolic space. As a corollary, we answer an old question of Steiner [Ste] in the classical subject of combinatorics of (Euclidean) polyhedra. We are now working to improve our techniques in order to make progress on the *Bending Measure Conjecture*, posed by Thurston in the hyperbolic setting and by Mess in the AdS setting.

Most recently, I have begun a new line of research in the subject of convex real projective structures [Be1, CLT, Gol]. While every hyperbolic structure is a convex projective structure, examples of convex projective structures on non-hyperbolic three-manifolds were found only recently by Benoist [Be2]. My recent joint work [BDL] with Sam Ballas and Gye-seon Lee uses deformation techniques to produce what we believe to be the largest known source of examples of convex projective structures on non-hyperbolic three-manifolds. As a corollary, the fundamental group of each such manifold admits a discrete faithful four-dimensional real matrix representation. Previous linearity results for these groups, obtained via cube complex techniques [PW], gave no control over the dimension of the representation. Our work suggests new theoretical and experimental techniques in the search for low-dimensional matrix representations of three-manifold groups.

[1] Independent work of Choi-Goldman [ChG] also proves the conjecture in a special case.

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