

⑩ See from (*) on in #9.

⑪ $I = \int \frac{\sqrt{t+9}}{t} dt$ $u = t+9$ $t = u-9$
 $du = dt$

$= \int \frac{\sqrt{u}}{u-9} du$ $w = \sqrt{u}$ $u = w^2$
 $du = 2w dw$

$= \int \frac{w}{w^2-9} \cdot 2w dw$

$= 2 \int \frac{w^2}{w^2-9} dw$

$= 2 \int \left(1 + \frac{9}{w^2-9} \right) dw$

$= 2w + 2 \cdot 3 \int \left(\frac{-1/2}{w+3} + \frac{1/2}{w-3} \right) dw$

$= 2w + 2 \cdot 3 \left(-\frac{1}{2} \ln|w+3| + \frac{1}{2} \ln|w-3| \right) + C$

$= 2w + 3 \ln \frac{|w-3|}{|w+3|} + C$

$= 2\sqrt{u} - 3 \ln \left| \frac{\sqrt{u}-3}{\sqrt{u}+3} \right| + C$

$= \boxed{2\sqrt{t+9} - 3 \ln \left| \frac{\sqrt{t+9}-3}{\sqrt{t+9}+3} \right| + C}$

$\frac{A}{w+3} + \frac{B}{w-3}$

$\frac{A(w-3) + B(w+3)}{w^2-9}$

$A+B=0$

$3B-3A=9$

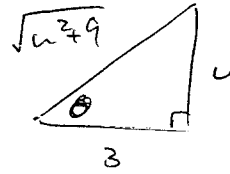
$B-A=3$

$A = -\frac{3}{2}$

$B = \frac{3}{2}$

$$\textcircled{11} \int \frac{\sqrt{t+9}}{t} dt \quad \begin{array}{l} u = \sqrt{t} \\ t = u^2 \\ dt = 2u du \end{array}$$

$$= \int \frac{\sqrt{u^2+9}}{u^2} \cdot 2u du = 2 \int \frac{\sqrt{u^2+9}}{u} du$$



$$= 6 \int \frac{\sec^3 \theta}{\tan \theta} d\theta = 6 \int \frac{1}{\sin \theta \cos^2 \theta} d\theta$$

$$= 6 \int \csc \theta \cdot \sec^2 \theta d\theta$$

$$= 6 \int \csc \theta \cdot (1 + \tan^2 \theta) d\theta$$

$$= 6 \int \csc \theta d\theta + 6 \int \sec \theta \tan \theta d\theta$$

$$= 6 \ln |\csc \theta - \cot \theta| + 6 \sec \theta + C$$

= :

$$\boxed{6 \ln \left| \frac{\sqrt{t+9}-3}{\sqrt{t}} \right| + 2\sqrt{t+9} + C} \left(= 3 \ln \left| \frac{\sqrt{t+9}-3}{\sqrt{t+9}+3} \right| + 2\sqrt{t+9} + C \right)$$

$$6 \ln \left| \frac{\sqrt{t+9}-3}{\sqrt{t}} \right| = 3 \ln \left| \frac{(\sqrt{t+9}-3)^2}{t} \right| = 3 \ln \left| \frac{(\sqrt{t+9}-3)(\sqrt{t+9}-3)(\sqrt{t+9}+3)}{t(\sqrt{t+9}+3)} \right|$$

$$= 3 \ln \left| \frac{(\sqrt{t+9}-3)(\cancel{t+9-9})}{t(\sqrt{t+9}+3)} \right| = 3 \ln \left| \frac{\sqrt{t+9}-3}{\sqrt{t+9}+3} \right|$$