M210T - Emerging Scholars Seminar
Worksheet 1
January 20, 2010

1. Find the following limits without using L'Hospital's Rule.

   a) $\lim_{x\to\infty} \frac{3x^2-5x}{5x^2+2x-6}$ b) $\lim_{x\to\infty} \left(\frac{2x-3}{3x+7}\right)^4$ c) $\lim_{x\to\infty} \left(\sqrt{x^2+x}-\sqrt{x^2+1}\right)$

   d) $\lim_{x\to\infty} \left(\sqrt[8]{x^2+1}-\sqrt[4]{x+1}\right)$ e) $\lim_{x\to\infty} \frac{1}{\pi}\int_{-x}^{x} \frac{1}{1+t^2}dt$

2. What does $\lim_{x\to\infty} f(x) = L$ mean?
   What does $\lim_{x\to c} f(x) = L$ mean?
   Prove that $\lim_{x\to\infty} \frac{1}{x} = 0$.
   Prove that $\lim_{x\to 2} x^2 = 4$.

3. Why is $\frac{0}{0}$ an indeterminate form?
   Are the following indeterminate forms? If not, what are limits of this form equal to?

   $$\frac{0}{\infty} \quad \frac{\infty}{\infty} \quad 0^0 \quad 0^\infty \quad \infty-\infty$$

   What are the other indeterminate forms?
   Find examples to verify each indeterminate form.

4. If $0 < a < b$, find $\lim_{t\to 0} \left(\int_0^1 [bx+a(1-x)]^t\,dx\right)^{1/t}$.

5. * A cake with dimensions $15\times 15\times 3$ is covered with a uniform layer of frosting. It's easy to cut the cake into 2 pieces so that each piece has the same amount of cake and the same amount of frosting. Can the cake be cut equally into 3 pieces? How about 5 pieces? How about n pieces? Can you cut a cake that is $10\times 20\times 3$ equally into n pieces?