

M210T - Emerging Scholars Seminar
Worksheet 10
March 3, 2010

1. For each parametric equation, write a Cartesian equation, and sketch a graph.

a) $x = 5 \cos t$ b) $x = 5 \sec t$ c) $x = \cos^3 t$
 $y = 3 \sin t$ $y = 3 \tan t$ $y = \sin^3 t$

d) $x = \log_{10} \theta$ e) $x = 10^s$ f) $x = t^m$
 $y = \ln \theta$ $y = e^s$ $y = t^n$

2. Graph each in the xy plane (they are all different). Show how each graph is traced as t varies.

a) $x = t$ b) $x = t^2$ c) $x = e^t$
 $y = t^2$ $y = t^4$ $y = e^{2t}$

d) $x = (1 - t^2)^{1/2}$ e) $x = t^{-1}$ f) $x = |t|^{1/2}$
 $y = 1 - t^2$ $y = t^{-1}$ $y = t$

3. Show that if x and y are functions of t , then

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$$

Find dy/dx two different ways for each parametric equation in exercise 1.

4. Can you find a way to find the arc length of a parametric curve $x = f(t)$, $y = g(t)$, $a \leq t \leq b$?

5. * Suppose we choose fifty-one integers from the set $\{1, 2, 3, 4, \dots, 100\}$.

a) Prove that there must be two integers among the fifty-one we chose that are relatively prime (that is, do not have a common prime factor).

b) Prove that there must be two integers among the fifty-one we chose such that one of the integers is a multiple of the other.