M210T - Emerging Scholars Seminar
Worksheet 13
March 29, 2010

1. Is the angle between the following vectors acute or obtuse: $\langle 3, 0, -1\rangle$ and $\langle -2, 4, 5\rangle$.

2. Find the vector projection of $4\mathbf{j} - 3\mathbf{k}$ onto $-2\mathbf{i} + \mathbf{j}$.

3. Find the equation of the plane that passes through the origin and is perpendicular to $\langle 2, 5, -2\rangle$.

4. Find the equation of the plane that contains the points $(1, 2, 3)$, $(4, 0, -1)$ and $(0, 0, 0)$.

5. Find the equation of the plane that contains the points $(1, 2, 3)$, $(4, 0, -1)$ and $(2, -2, -3)$.

6. Find all values of c for which $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $-4\mathbf{i} + c\mathbf{j} + \mathbf{k}$, and $3\mathbf{j} + c\mathbf{k}$ are coplanar.

7. Find the orthogonal projection of $\langle 3, 2, 1\rangle$ onto the plane $x - 2y + 4z = 0$ (you of course will need to determine what I mean by *orthogonal projection*).

8. Find the area of the parallelogram defined by the vectors $\langle 1, -1, 1\rangle$ and $\langle 2, -1, -1\rangle$.

9. Find a nonzero vector that lies in each of the planes with the following equations: $x + y + 3z = 0$ and $4x - z = 3$.

10. Find an equation of the line in $\mathbb{R}^3$ that contains the points $(3, 2 - 1)$ and $(4, -8, 3)$.

11. * Which is larger between the set of integers and the set of even integers (what does it mean for one infinite set to be larger than another?)?
    What about the set of even integers and the set of odd integers?
    What about the set of natural numbers, $\{1, 2, 3, \ldots\}$, and the set of integers?
    What about the set of natural numbers and the set of rational numbers? (Hint: all of the sets above are the same size).
    What about the set of real numbers and the set of natural numbers?

    If a set is the same size as the set of natural numbers, the set is said to be *countably infinite*, and its cardinality (the number of elements it contains) is $\aleph_0$. If the set is larger, it is said to be *uncountable*. For example, the set of real numbers is uncountable, and its cardinality is $\aleph_1$.