M210T - Emerging Scholars Seminar
Worksheet 14
March 31, 2010

1. Find the intersection of and angle between the following planes:

$$x - 2x + 4z = 9 \quad \text{and} \quad 2x + y - 2z = -2.$$

2. Find a line in the plane $3x - 4z + 5 = 0$.

3. Are the following lines coplanar? If so, find an equation for the plane.

$$\mathbf{r}(t) = \langle 3, -2, 1\rangle + \langle 1, 1, 1\rangle t \quad \text{and} \quad \mathbf{r}(t) = \langle 1 + t, -4 - t, -1 + 3t\rangle.$$

What about

$$\mathbf{r}(t) = \langle 3, -2, 1\rangle + \langle 1, 1, 1\rangle t \quad \text{and} \quad \mathbf{r}(t) = \langle 1 + t, -4 + t, -1 + t\rangle.$$

What about

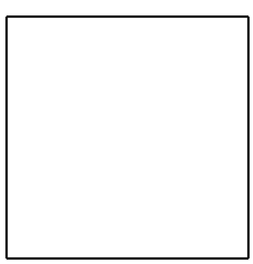
$$\mathbf{r}(t) = \langle 3 - t, -2 + t, 1 + t\rangle \quad \text{and} \quad \mathbf{r}(t) = \langle 1 + t, -4 - t, -1 + 3t\rangle.$$

4. Find the orthogonal projection of the line defined by $\frac{x-3}{2} = \frac{y+1}{5} = \frac{z}{-4}$ onto the plane defined by $x - y + z = 8$.

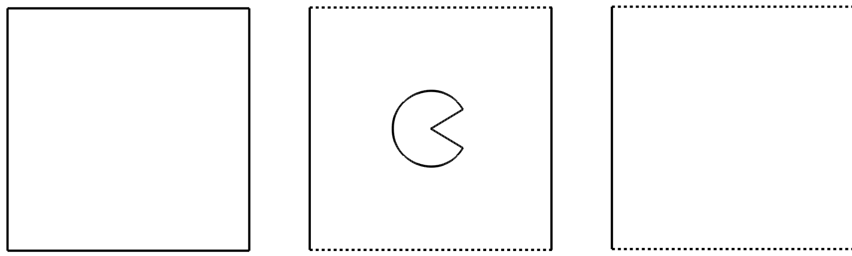
5. Consider the function $\mathbf{r}(t) = \langle 2t^2, 3t^3 - 1, 5e^t\rangle$. What kind of function is this (i.e., what are the domain and range, and what might a picture of it look like?)? How do you think we would define the derivative of this function? Find the line tangent to this graph at $\mathbf{r}(0)$.

6. Consider the function $f(x, y) = 3x + 4y - 8$. What would the plot of this look like? How do you think we might define a derivative for this kind of function? Can we talk about a tangent line, or does something else make more sense? Now consider the function $f(x, y) = 4x^2 + 5y^2$. Discuss its "derivative" and tangent stuff at the point $(0, 0, -8)$.

7. * The game *Asteroids* takes place in a weird world. If an asteroid or the ship leaves the right side of this universe, it comes back through the left side, and if it leaves the top of the universe, it comes back through the bottom. Another way to think of this is, in this world, moving up the right side of the universe is the same as moving up the left side, and moving across the top of this world is the same as moving across the bottom. We can illustrate this with the picture below (Pac-Man's world is on the back):



What three-dimension object does this space look like? What about the following spaces, described similarly (these questions lead to the study of algebraic topology):



These are a little harder: