1. Find the unit tangent vectors and tangent lines to the following curves at $t = 0$:

$$r(t) = \cos t \mathbf{i} + 3t \mathbf{j} + 2\sin(2t) \mathbf{k}$$

$$f(t) = (te^{-t}, 2\tan^{-1} t, 2e^t)$$

$$g(t) = e^t \mathbf{i} + te^t \mathbf{j} + te^t \mathbf{k}$$

2. Find the length of the following curves:

$$2t \mathbf{i} + \frac{4t^{3/2}}{3} \mathbf{j} + \frac{t^2}{2} \mathbf{k}, \quad 0 \leq t \leq 1$$

$$\langle 3t^2, 3t, 4t^2 \rangle, \quad 0 \leq t \leq 2$$

3. Consider the function $f(t) = (\sin t, \cos t, t)$.  

What does this look like?  
Find the angle the curve makes with the plane $z = 0$.  
Find the angle the curve makes with the plane $z = \pi$.  
Find the length of the curve between the two planes, $z = 0$ and $z = \pi$.

4. Consider the function $f(x, y) = 3x + 4y - 8$. What would the plot of this look like? How do you think we might define a derivative for this kind of function? Can we talk about a tangent line, or does something else make more sense? Now consider the function $f(x, y) = 4x^2 + 5y^2$. Discuss its "derivative" and tangent stuff at the point $(0, 0, -8)$.

5. * Let $C([0, 1])$ be the space of continuous functions on the interval $[0, 1]$. Much like $\mathbb{R}$ or $\mathbb{R}^3$, this is a vector space (because sums and multiples of things in $C([0, 1])$ are still in $C([0, 1])$, distribution laws hold, and there is a zero function $0(x) = 0$). Can you think of an inner product for this space? Recall that an inner product satisfies the following conditions: 

a) $(f, g) = (g, f)$ for all $f, g \in C([0, 1])$

b) $(kf, g) = k(f, g)$ for all $f, g \in C([0, 1])$, $k \in \mathbb{R}$.

c) $(f + g, h) = (f, h) + (g, h)$ for all $f, g, h \in C([0, 1])$.

d) $(f, f) \geq 0$ for all $f \in C([0, 1])$, and $(f, f) = 0$ if and only if $f = 0$ (i.e. $f(x) = 0$ for all $x \in [0, 1]$).

Find two functions that are orthogonal. Find the norm of the function $f(x) = x$. 

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Footnote:

* The notation $C([0, 1])$ represents the space of continuous functions on the interval $[0, 1]$. This space is often referred to as the space of continuous functions on $[0, 1]$. It is a vector space because it contains the zero function, and the operations of addition and scalar multiplication are well-defined. The space $C([0, 1])$ also has an inner product defined by $(f, g) = \int_0^1 f(x)g(x)\,dx$ for $f, g \in C([0, 1])$. This inner product satisfies the conditions mentioned in the footnote. The notation $C([0, 1])$ is often used to denote the space of continuous functions on $[0, 1]$. The space $C([0, 1])$ is a vector space because it contains the zero function, and the operations of addition and scalar multiplication are well-defined. The space $C([0, 1])$ also has an inner product defined by $(f, g) = \int_0^1 f(x)g(x)\,dx$ for $f, g \in C([0, 1])$. This inner product satisfies the conditions mentioned in the footnote.