M210T - Emerging Scholars Seminar
Worksheet 15
April 5, 2010

1. Find the unit tangent vectors and tangent lines to the following curves at $t = 0$:

$$\mathbf{r}(t) = \cos t\mathbf{i} + 3t\mathbf{j} + 2\sin(2t)\mathbf{k}$$
$$\mathbf{f}(t) = \langle te^{-t}, 2\tan^{-1} t, 2e^t\rangle$$
$$\mathbf{g}(t) = e^t\mathbf{i} + te^t\mathbf{j} + te^{t^2}\mathbf{k}$$

2. Find the length of the following curves:

$$2t\mathbf{i} + \frac{4t^{3/2}}{3}\mathbf{j} + \frac{t^2}{2}\mathbf{k}, \quad 0 \le t \le 1$$
$$\langle 3t^2, 3t, 4t^2\rangle, \quad 0 \le t \le 2$$

3. Consider the function $\mathbf{f}(t) = \langle \sin t, \cos t, t\rangle$.
   What does this look like?
   Find the angle the curve makes with the plane $z = 0$.
   Find the angle the curve makes with the plane $z = \pi$.
   Find the length of the curve between the two planes, $z = 0$ and $z = \pi$.

4. Consider the function $f(x, y) = 3x+4y-8$. What would the plot of this look like? How do you think we might define a derivative for this kind of function? Can we talk about a tangent line, or does something else make more sense? Now consider the function $f(x, y) = 4x^2 + 5y^2$. Discuss its "derivative" and tangent stuff at the point $(0, 0, -8)$.

5. * Let $\mathcal{C}([0, 1])$ be the space of continuous functions on the interval $[0, 1]$. Much like $\mathbb{R}$ or $\mathbb{R}^3$, this is a *vector space* (because sums and multiples of things in $\mathcal{C}([0, 1])$ are still in $\mathcal{C}([0, 1])$, distribution laws hold, and there is a *zero* function $\mathbf{0}(x) = 0$). Can you think of an inner product for this space? Recall that an inner product satisfies the following conditions: a) $(f, g) = (g, f)$ for all $f, g \in \mathcal{C}([0, 1])$
   b) $(kf, g) = k(f, g)$ for all $f, g \in \mathcal{C}([0, 1])$, $k \in \mathbb{R}$.
   c) $(f + g, h) = (f, h) + (g, h)$ for all $f, g, h \in \mathcal{C}([0, 1])$.
   d) $(f, f) \ge 0$ for all $f \in \mathcal{C}([0, 1])$, and $(f, f) = 0$ if and only if $f = \mathbf{0}$ (i.e. $f(x) = 0$ for all $x \in [0, 1]$).

   Find two functions that are orthogonal. Find the *norm* of the function $f(x) = x$.