M210T - Emerging Scholars Seminar
Worksheet 16
April 12, 2010

1. Find the first and second partial derivatives of the following functions and verify Clairaut's Theorem.

$$f(x,y) = e^{x^2 y}, \qquad z = (x^3 + 2xy - y)^3, \qquad g(x,y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

2. Which would increase the volume of a can more: an increase in the height, or an increase in the radius of the base? Which would you decrease for a larger decrease in volume?

3. Suppose that the function $z = f(x,y)$ is continuous. Let's determine what the function looks like near the point $(a, b, f(a,b))$.

   a) If you were to talk about an object being tangent to the surface at the point $(a, b, f(a,b))$, what would this object be? In other words, if you were to zoom in very close to the point $(a, b, f(a,b))$, what does the surface resemble?

   b) Find a line that is tangent to the surface that has x fixed as $x = a$. To do this, consider the curve (i.e. the parametric function) that lies in the surface with $x = a$. The equation of this curve would look like

$$\mathbf{r}(t) = \langle a, t, f(a,t) \rangle.$$

   The line tangent to this curve at the point $(a, b, f(a,b))$ would be tangent to the surface and have $x = a$.

   c) Similarly, determine a line that is tangent to the surface that has y fixed as $y = b$.

   d) Can you use these lines to talk about the behavior of f near $(a, b, f(a,b))$?

4. Let $f(x,t) = A\cos(x + \omega t) + B\sin(x + \omega t)$, and show that f satisfies the wave equation (in one spatial dimension):

$$\frac{\partial^2 f}{\partial t^2} = \omega^2 \frac{\partial^2 f}{\partial x^2}.$$

   Can you find a function $g(x,y,t)$ that satisfies the wave equation in two spatial dimensions:

$$\frac{\partial^2 g}{\partial t^2} = \omega^2 \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)?$$

5. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^y}{x^2+y^2} = 0$ and that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exist.

6. * Show that $\sqrt{2}$ is irrational. Can you extend your argument to show that $\sqrt{p}$ is irrational for any prime number p?