M210T - Emerging Scholars Seminar
Worksheet 17
April 14, 2010

1. Find the plane that is tangent to the following functions at the given points:

$$z = \sin^2(x) + \cos^2(y) \quad \text{at} \quad (\pi, \pi/2),$$
$$f(x,y) = e^{xy^2} \quad \text{at} \quad (0,0),$$
$$f(x,y) = \frac{x}{xy+y} \quad \text{at} \quad (2,1).$$

2. The total resistance R of three resistors in parallel is given by the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3},$$

where R_1, R_2, and R_3 are the resistances of the three resistors. If the three resistors are variable resistors,

a) find the linearization of the total resistance when the resistors are set at $R_1 = 100\ \Omega$, $R_2 = 150\ \Omega$, and $R_3 = 300\ \Omega$ ($1\ \Omega$ is an *Ohm*, the SI unit of resistance).

b) find how fast the total resistance is changing if each resistor is increasing at a rate of $2\ \Omega$ per minute.

3. a) If you are standing on the plane $z = ax + by + c$, in which direction should you move on the surface so that the height is increasing at the fastest rate (your answer may be in terms of the constants a, b, c)?

b) If you are standing on the differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ at the point $(x_0, y_0, f(x_0, y_0))$, in which direction should you move so that the height is increasing at the fastest rate (your answer may be in terms of x_0, y_0, and f and it's derivatives)?

c) How fast does the height of the surface in (a) increase if you move in the direction $\langle u, v \rangle$? What about the function in (b)?

4. Find the second order Taylor approximation of the function $f(x,y) = e^{xy^2}$ at the point $(0,0)$. What does this look like graphically?

5. * Show that there are infinitely many prime numbers.