M210T - Emerging Scholars Seminar
Worksheet 18
April 19, 2010

1. Define $f(x, y) = x^2 + y^2 + 4y$.

   a. What are the first and second derivatives of f?

   b. Does f have any local extrema? If so, where are they?

   c. What is the gradient of f at the point (x, y)? Sketch the gradient field.

   d. What is the directional derivative of f at the point $(1, 2)$ in the direction $\langle 12, -5\rangle$?

   e. Consider the surface given by the equation $z = f(x, y)$. If you placed a ball at the point $(1, 1, 6)$ on this surface, in what direction would it begin to roll?

2. What point on the ellipse given by the equation $4x^2 + 9y^2 = 36$ is farthest from the point $(5, 5)$? What point on the ellipse is closest to the line $x + 7y = 35$?

3. Use the gradient to find a normal vector to the surface given by the equation $x^2 + y^2 - z^2 = 1$ at the point (7.4.8). Use this to find the tangent plane at this point. (You should be able to do this without solving for z).

4. Suppose you want to maximize the function $f(x, y)$ given the restriction that $g(x, y) = 0$. How might you approach this problem using gradients?

5. * Spot the error in this proof by induction that all sheep are the same color:

   Let $P(n)$ be the statement: Any set of n sheep are all the same color.
   $P(1)$ is obviously true.

   Let A contain n sheep. Construct B using all the sheep from A, except swap one sheep in A for a different one (call this new sheep S) not from A.

   If $P(n)$ is true, then both A and B contain sheep of the same colour, since they both have n sheep.

   Now, S is the same color as all other sheep in B. But all other sheep in A are also in A, so S is also the same color as all sheep in A. So both B and A contain sheep with the same color. If we re-insert S into A, we get a new set with $n + 1$ sheep, all of the same colour. So $P(n + 1)$ is true.

   $P(1)$ is true, $P(n + 1)$ follows from $P(n)$, so $P(n)$ is true for all n.