M210T - Emerging Scholars Seminar
Worksheet 19
April 26, 2010

1. Find and identify (if possible) the critical points of the following functions:

$$f(x,y) = xye^{xy}, \qquad g(x,y) = 3x - x^3 - 2y^2 + y^4$$

2. Consider the triangle with vertices $A(0,0)$, $B(0,30)$, and $C(15,0)$. Find the points Q on the triangle (on the interior or boundary) maximize or minimize the value of $|\overrightarrow{AQ}|^2 + |\overrightarrow{BQ}|^2 + |\overrightarrow{CQ}|^2$.

3. Find a function f so that $f_x(0,0) = f_y(0,0) = 0$, f_{xx} and f_{yy} are both positive, and f does not have a local minimum at $(0,0)$.

4. What point on the ellipse given by the equation $4x^2 + 9y^2 = 36$ is farthest from the point $(5,5)$? What point on the ellipse is closest to the line $x + 7y = 35$?

5. Suppose you want to maximize the function $f(x,y)$ given the constraint that $g(x,y) = 0$. How might you use gradients to approach this? Take for example $f(x,y) = 9x^2 + 16y^2$ given the constraint that $g(x,y) = x^2 + y^2 - 4 = 0$.

6. * You create a painting using only three colors: red, white, and blue. Prove or disprove that there must be two points on the painting that are exactly one inch apart that are the same color.