M210T - Emerging Scholars Seminar
Worksheet 2
January 25, 2010

1. Which of the following are improper integrals? Why?

$$\int_4^\infty \frac{1}{x^2}\, dx, \quad \int_0^3 \frac{1}{\left(\frac{1}{x-2}\right)}\, dx, \quad \int_0^\pi \tan x\, dx, \quad \int \frac{1}{x}\, dx, \quad \int_{1/e}^e \frac{1}{x \ln x}\, dx, \quad \int_0^1 x^p \ln x\, dx.$$

Compute some of them.

2. a) Recall that $\lim\limits_{x\to\infty} f(x) = L$ (where L is a real number) means given some $\epsilon > 0$ (we usually think of this as an arbitrarily small number; i.e. as small as you want to pick it), there is an $N > 0$ (which may depend on ϵ) so that if $x > N$, then $|f(x) - L| < \epsilon$. Write down what it means for $\int_0^\infty g(x)\, dx = L$.

b) Recall that $\lim\limits_{x\to c^+} f(x) = L$ (where L and c are real numbers) means given some $\epsilon > 0$, there is a $\delta > 0$ so that if $0 < x - c < \delta$, then $|f(x) - L| < \epsilon$. Write down what it means for $\int_0^4 x^{-1/2}\, dx = L$. What is L in this case?

3. Trent and Cassie are discussing the improper integral $\int_{-\infty}^\infty \frac{x}{1+x^2}\, dx$. Trent says the integral diverges, but Cassie says that the integral is 0 because the function $f(x) = \frac{x}{1+x^2}$ is odd. Are either of them correct?

4. Find a function $f(x)$ with the following properties:
   a) f is differentiable on the interval $(0, \infty)$.
   b) $f(x) > 0$.
   c) $\lim_{x\to 0^+} = \infty$.
   d) $\int_0^\infty f(x)\, dx$ converges.
   You don't need to be able to compute the integral in d, but bonus points if you can.

5. Show that $\int_1^\infty \frac{1}{x\left(\sqrt{\ln x} + \ln^2 x\right)}\, dx$ converges. It is probably easiest to do this without actually computing the integral (how can you do this?). However, computing the integral would be a good review of integration techniques. You may want to do that outside of class though, because it will probably take a while.

6. * A woman has exactly two children. At least one of them is a boy who was born at 1:30 a.m. (meaning between 1:30:00 a.m. and 1:31:00 a.m.). What is the probability that she has two sons?