**M210T - Emerging Scholars Seminar**
**Worksheet 20**
**April 28, 2010**

1. What point on the ellipse given by the equation $4x^2 + 9y^2 = 36$ is farthest from the point $(5, 5)$? What point is closest?

2. Find the extreme values of the function $f(x, y) = e^{-xy}$ in the right half plane bounded by $x^2 + 4y^2 \leq 1$.

3. Suppose we are trying to minimize the function $f(x, y) = x$ on the curve $y^2+x^4-x^3 = 0$.

   a) Try using Lagrange multipliers to solve this problem.

   b) Show that the minimum value of $f$ on the curve actually occurs at $(0, 0)$.

   c) Why does (b) not agree with (a)? i.e. Why does the method of Lagrange multipliers fail in this case?

4. Suppose that $x_1, x_2, \ldots, x_n$ are nonnegative real numbers so that $\sum_{k=1}^{n} x_k = c$. Find the minimum and maximum values of $\sqrt[n]{x_1 x_2 \cdots x_n}$. Use your result to compare the geometric and arithmetic means of the $x_k$'s.

5. What point on the ellipse in number 1 is closest to the line $x + 7y = 35$?

6. * The probability that a women between 40 and 50 years old has breast cancer is 0.8 percent. If a woman has breast cancer, the probability is 90 percent that she will have a positive mammogram. If a woman does not have breast cancer, the probability is 7 percent that she will still have a positive mammogram. Imagine a woman who has a positive mammogram. What is the probability that she actually has breast cancer? (In a study, 95 out of 100 American doctors estimated incorrectly).