M210T - Emerging Scholars Seminar
Worksheet 21
May 3, 2010

1. Compute the following integrals:

$$\int_0^1 \int_0^1 \sqrt{x+y}\; dx\; dy$$
$$\iint_R xye^{x^2y}\; dA, \quad R = [0,1] \times [0,2]$$
$$\int_1^2 \int_0^1 \frac{t}{s^2+t^2}\; ds\; dt.$$

2. Compute the following integrals:

$$\int_1^3 \int_y^{y^2} xy\; dx\; dy$$
$$\iint_D y \sin x^3\; dA, \quad D = \{(x,y) \mid 0 \le x \le \pi, x \le y \le 2x\}$$
$$\iint_D (x+y)\; dA, \quad D \text{ is bounded by } y = \sqrt{x} \text{ and } x = \sqrt{y}.$$

3. Given that f is a continuous function, switch the order of integration on the following integrals:

$$\int_0^1 \int_0^x f(x,y)\; dx\; dy \qquad \int_0^1 \int_{y^2}^y f(x,y)\; dx\; dy \qquad \int_{-2}^2 \int_{x^2}^4 f(x,y)\; dx\; dy \qquad \int_0^{\pi/2} \int_0^{\tan x} f(x,y)\; dy\; dx.$$

4. Compute the integral
$$\int_1^\infty \int_1^y \frac{1}{x^4+x^2y^2}\; dx\; dy.$$

5. Notice that the area of a "polar rectangle" defined by $\{(r,\theta) \mid r_1 \le r \le r_2, \theta_1 \le \theta \le \theta_2\}$ depends on how large r_1 and r_2 are (as opposed to a typical rectangle, whose area only depends on the *differences* between the x's and y's). Show that the area is $\bar{r}\Delta r \Delta\theta$ where $\bar{r} = \frac{r_1+r_2}{2}$, $\Delta r = r_2 - r_1$ and $\Delta\theta = \theta_2 - \theta_1$.

6. $\frac{*}{2}$ (I wasted a few hours of my weekend doing this problem a very long and tedious way. There is, of course, a quick way that I didn't think of until this morning. Here's an opportunity for you to feel smarter than your instructor, if you don't already.) Compute

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^3}{n^4} \cos\left(\pi \frac{k^2}{n^2}\right).$$

7. * What is wrong with the following proof that $3 = 0$?
Assume that
$$x^2 + x + 1 = 0.$$
Then
$$x^2 = -x - 1.$$
Since 0 is clearly not a solution, we can divide by x and get
$$x = -1 - \frac{1}{x}.$$
Substituting this in place of the linear term of the original equation yields
$$x^2 + \left(-1 - \frac{1}{x}\right) + 1 = 0$$
$$x^2 - \frac{1}{x} = 0$$
$$x^2 = \frac{1}{x}$$
$$x^3 = 1$$
$$x = 1.$$
Substituting into the original equation gives us $3 = 0$.