**M210T - Emerging Scholars Seminar**
**Worksheet 3**
**January 27, 2010**

1. Do the following sequences converge or diverge? If they converge, what do they converge to?

$$a_n = \sqrt{n}\sin(\pi/\sqrt{n}), \qquad b_n = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{(2n)^n},$$

$$\left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}\right\}, \qquad d_n = \left(1+\frac{a}{n}\right)^n,$$

$$e_n = \frac{\cos(n)}{n}, \qquad f(n) = \begin{cases} 1 \text{ if } n = 2^k \text{for some integer } k \\ 0 \text{ otherwise} \end{cases}.$$

2. Write down a definition (without using words like "close" or "approaches") for $\lim_{n\to\infty} a_n = L$. Use this definition to prove that $a_n = \frac{3+n}{2n}$ converges to $\frac{1}{2}$ and $b_n = (-1)^n$ diverges.

3. Define the sequence $\{a_n\}$ by $a_1 = 1$ and $a_{n+1} = 1 + \frac{1}{a_n}$ for $n \geq 1$. Go ahead and assume that $a_n$ converges. What does it converge to?

4. The Fibonacci Sequence is defined recursively by

$$F_1 = F_2 = 1, \quad F_n = F_{n-1} + F_{n-2} \text{ for } n \geq 3.$$

Consider the sequence defined by $a_n = \frac{F_{n+1}}{F_n}$. Assume that $a_n$ converges, and figure out what it converges to.

5. Prove that $a_n = \dfrac{(2n-1)!}{n^{2n-1}}$ converges.

6. Prove that $a_n = \cos(n)$ diverges.

7. * You have nine marbles of the same diameter. Eight of them have the same weight, and one is heavier than the others. You also have a balance. Can you determine which marble is the heavy one using the balance only twice? If you have $n$ marbles, with one heavier than the others, what is the minimum number of weighings that would guarantee that you can find the heavy one?

8. * Show that there must be two people in this class who know the same number of people from this class (note that knowing someone is reflexive; i.e. you know me if and only if I know you).