M210T - Emerging Scholars Seminar
Worksheet 4
February 1, 2010

1. Do the following series converge or diverge? Why? If they converge, what do they converge to?

$$\sum_{n=0}^{\infty} 7\left(\frac{2}{5}\right)^n, \quad \sum_{n=1}^{\infty} \tan^{-1} n, \quad \sum_{k=1}^{\infty} \ln\left(\frac{k}{2k+5}\right), \quad \sum_{i=1}^{\infty} \frac{1}{i+3}, \quad \sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$$

2. Write down a definition for convergence of a series; i.e. what does it mean for the series $\sum_{n=0}^{\infty} a_n$ to converge? (by now you know we want a precise definition).
Use this definition to prove that $\sum_{n=0}^{\infty}(-1)^n$ diverges and $\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$ converges.

3. I claim that the series $\sum_{n=0}^{\infty}(-2)^n$ converges. Here's my argument:

Let $L = \sum_{n=0}^{\infty}(-2)^n$. Then

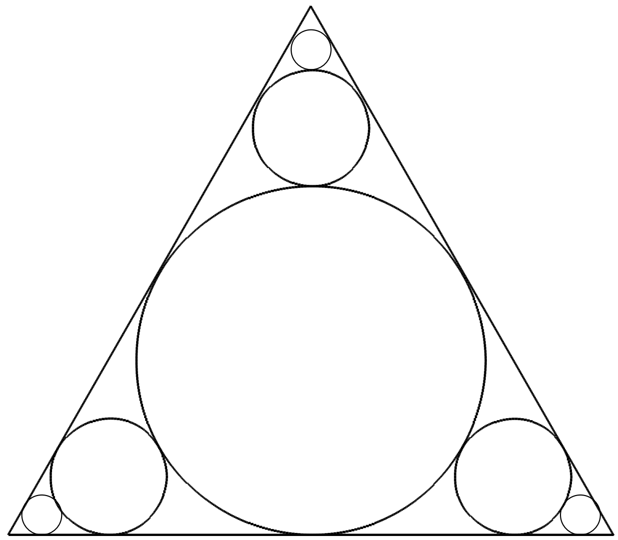
$$\begin{aligned} L &= \sum_{n=0}^{\infty}(-2)^n \\ &= 1 - 2 + 2^2 - 2^3 + \ldots \\ &= 1 - 2\left(1 - 2 + 2^2 - \ldots\right) \\ &= 1 - 2\sum_{n=0}^{\infty}(-2)^n \\ &= 1 - 2L. \end{aligned}$$

Solving $L = 1 - 2L$, we have $L = \frac{1}{3}$. Thus $\sum_{n=0}^{\infty}(-2)^n = \frac{1}{3}$.

Why does this series converge despite being a geometric series with $r = -2$? Or have I made a mistake?

4. An infinite geometric series consisting of positive terms has the property that the second term of the series is the square root of the first term. What is the smallest possible sum of this series?

5. In the figure below, there are infinitely many circles approaching the vertices of an equilateral triangle, and each circle touches other circles and sides of the triangle. Find the total area occupied by the circles.



6. * A cube of cheese has been subdivided into 27 subcubes. A mouse starts to eat a corner subcube. After eating any given subcube it goes on to another adjacent subcube. Is it possible for the mouse to eat all 27 subcubes and finish with the center cube?