M210T - Emerging Scholars Seminar
Worksheet 8
February 24, 2010

Note: Many of these problems are duplicates from the previous worksheet.

1. Find a power series centered at $a = 0$ for each of the following functions. Note that since each series derives from the first, each has a radius of convergence of $R = \underline{\ ?\ }$:

$$\frac{1}{1-x}, \quad \frac{1}{2+x}, \quad \ln(1-x), \quad \frac{1}{(1-x)^2}, \quad \frac{1}{1-x^2}, \quad \frac{x^4}{(1-x)^2}, \quad \frac{1}{(1-x)^3}, \quad \tan^{-1} x$$

2. Find a power series centered at $a = 0$ for each of the following functions, and determine the radius of convergence.

$$\ln\left(\frac{1-x}{1+x}\right), \quad \int_0^x \frac{x}{1+8x^3} dx, \quad \frac{1}{2x^2+5x+2}$$

3. Find two different power series for both $\ln(x)$ and $1/x$.

4. On Worksheet 7, number 6, a few students told me that the expected (average) number of moves the spider would make is

$$\sum_{n=1}^{\infty} \frac{2n}{2^n}.$$

We can easily show that this converges (e.g. by using the Ratio Test), but can you determine without a doubt what it converges to? (This problem turned out to be more timely than I thought.)

5. Express the following definite integrals as a series (I expect questions...).

$$\int_0^{1/2} x^3 \tan^{-1}(x^2) dx, \quad \int_0^{1/3} \frac{x^2}{1+x^4} dx, \quad \int_0^2 \frac{3x}{1-2x^2+x^4}$$

6. (This problem was on Worksheet 7, but I meant to put it here. This should lead into Friday's lecture.)
Suppose that the function f has a power series representation at a with $R > 0$; i.e., on some interval,

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n.$$

- Find c_0 in terms of a value of f (feel free to ask for a hint).
- Write a power series for $f'(x)$. Use this to find c_1.

- Write a power series for $f''(x)$. Use this to find c_2.
- Find a general formula for $f^{(n)}(x)$. Use this to find c_n.

What does this tell you about the coefficients b_n and c_n if $\sum b_n(x-a)^n = \sum c_n(x-a)^n$?

7. (Speaking of Friday's lecture, here are some problems to consider once you've completed the previous problem or after Friday's lecture.)
   Find the power series centered at $a = 0$ for the following functions:
$$\cos x, \quad e^x, \quad \int_0^x e^{t^2}\,dt, \quad x \sin x$$

8. * Suppose you're in a hallway lined with 100 closed lockers. You begin by opening every locker. Then you close every second locker. Then you go to every third locker and open it (if it's closed) or close it (if it's open). Let's call this action toggling a locker. Continue toggling every nth locker on pass number n. After 100 passes, where you toggle only locker #100, how many lockers are open?