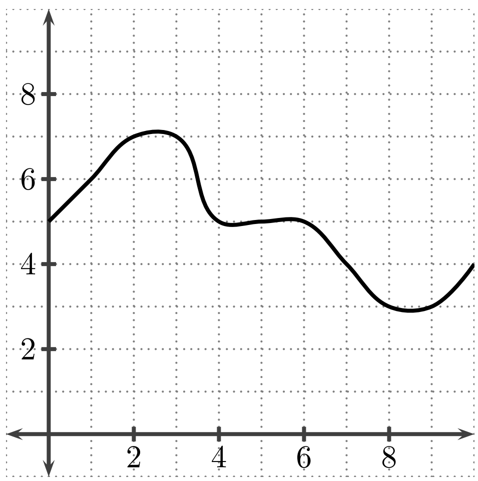
This print-out should have 16 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering. V1:1, V2:1, V3:3, V4:1, V5:1.

001 (part 1 of 1) 10 points

If f is the function whose graph on [0, 10] is given by



use the Trapezoidal Rule with $n = 5$ to estimate the definite integral

$$I = \int_3^8 f(x)\,dx\,.$$

1. $I \approx 23$

2. $I \approx \frac{47}{2}$

3. $I \approx 22$

4. $I \approx 24$ **correct**

5. $I \approx \frac{45}{2}$

Explanation:

The Trapezoidal Rule estimates the definite integral

$$I = \int_3^8 f(x)\,dx$$

by

$$I \approx \frac{1}{2}\Big[f(3) + 2\{f(4) + \cdots + f(7)\} + f(8)\Big]$$

when $n = 5$. For the given f, therefore,

$$\boxed{I \approx \frac{1}{2}\Big[7 + 2\{5 + 5 + 5 + 4\} + 3\Big] = 24},$$

reading off the values of f from the graph.

keywords: trapezoidal rule, integral, graph

002 (part 1 of 1) 10 points

Find

$$\int \frac{e^{7x}}{9 + e^{14x}}\,dx\,.$$

1. $\frac{1}{3}\arcsin e^{7x} + C$

2. $\arcsin e^{7x} + C$

3. $\frac{1}{21}\arctan\left(\frac{1}{3}e^{7x}\right) + C$ **correct**

4. $\frac{1}{3 + e^{7x}} + C$

5. None of these.

6. $\frac{1}{3}\text{arcsec}\, e^{7x} + e^{7x} + C$

Explanation:

$$\begin{aligned}\int \frac{e^{7x}}{9 + e^{14x}}\,dx &= \frac{1}{7}\int \frac{7e^{7x}\,dx}{(3)^2 + (e^{7x})^2} \\ &= \frac{1}{7}\int \frac{d(e^{7x})}{(3)^2 + (e^{7x})^2} \\ &= \frac{1}{7}\cdot\frac{1}{3}\arctan\left(\frac{1}{3}e^{7x}\right) + C \\ &= \frac{1}{21}\arctan\left(\frac{1}{3}e^{7x}\right) + C\end{aligned}$$

keywords: exponential function, inverse trig function

003 (part 1 of 1) 10 points

Evaluate the definite integral

$$I = \int_0^1 \frac{x-11}{x^2-x-2}\,dx\,.$$

1. $I = 7\ln 3$

2. $I = \ln 3$

3. $I = -7\ln 3$

4. $I = 7\ln 2$ **correct**

5. $I = -\ln 3$

6. $I = \ln 2$

7. $I = -7\ln 2$

8. $I = -\ln 2$

Explanation:

After factorization

$$x^2 - x - 2 = (x+1)(x-2)\,.$$

But then by partial fractions,

$$\frac{x-11}{x^2-x-2} = \frac{4}{x+1} - \frac{3}{x-2}\,.$$

Now

$$\int_0^1 \frac{4}{x+1}\,dx = \Big[4\ln|(x+1)|\Big]_0^1 = 4\ln 2\,,$$

while

$$\int_0^1 \frac{3}{x-2}\,dx = \Big[3\ln|(x-2)|\Big]_0^1 = -3\ln 2\,.$$

Consequently,

$$\boxed{I = 7\ln 2}\,.$$

keywords: definite integral, rational function, partial fractions, natural log

---

004 (part 1 of 1) 10 points

Evaluate the definite integral

$$I = \int_0^{\pi/4} \frac{4\cos x + 6\sin x}{\cos^3 x}\,dx\,.$$

1. $I = \frac{11}{2}$

2. $I = \frac{5}{2}$

3. $I = 10$

4. $I = 7$ **correct**

5. $I = 1$

Explanation:

After division we see that

$$\frac{4\cos x + 6\sin x}{\cos^3 x} = 4\sec^2 x + 6\tan x\sec^2 x = (4 + 6\tan x)\sec^2 x\,.$$

Thus

$$I = \int_0^{\pi/4} (4 + 6\tan x)\sec^2 x\,dx\,.$$

Let $u = \tan x$; then

$$du = \sec^2 x\,dx\,,$$

while

$$x = 0 \implies u = 0\,,$$
$$x = \frac{\pi}{4} \implies u = 1\,.$$

In this case

$$I = \int_0^1 (4 + 6u)\,du = \big[4u + 3u^2\big]_0^1\,.$$

Consequently,

$$\boxed{I = 7}\,.$$

keywords: trig integral, trig identity

005 (part 1 of 1) 10 points

Determine the indefinite integral

$$I = \int \frac{1}{\sqrt{6x - x^2}}\, dx\,.$$

1. $I = \sin^{-1}\Big(\dfrac{x+3}{3}\Big) + C$

2. $I = 2\sqrt{6x - x^2} + C$

3. $I = \dfrac{1}{3}\sin^{-1}(x-3) + C$

4. $I = \sin^{-1}\Big(\dfrac{x-3}{3}\Big) + C$ **correct**

5. $I = \dfrac{\sqrt{6x - x^2}}{3} + C$

Explanation:

After completing the square we see that

$$\begin{aligned} x^2 - 6x &= (x^2 - 6x + 9) - 9 \\ &= (x-3)^2 - 9\,. \end{aligned}$$

In this case

$$I = \int \frac{1}{\sqrt{9 - (x-3)^2}}\, dx\,.$$

To evaluate this last integral, set

$$x - 3 = 3\sin\theta\,.$$

For then

$$dx = 3\cos\theta\, d\theta\,,$$

while

$$\sqrt{9 - (x-3)^2} = 3\cos\theta\,,$$

in which case

$$I = \int \frac{3\cos\theta}{3\cos\theta}\, d\theta = \int d\theta\,.$$

Consequently,

$$\boxed{I = \theta + C = \sin^{-1}\Big(\frac{x-3}{3}\Big) + C}$$

with C an arbitrary constant.

keywords:

006 (part 1 of 1) 10 points

Determine the value of the definite integral

$$I = \int_6^{12} \frac{\ln t}{t^2}\, dt.$$

1. $I = \dfrac{1}{6} + \dfrac{1}{6}\ln 3$

2. $I = \dfrac{1}{12} + \dfrac{1}{3}\ln 3$

3. $I = \dfrac{1}{6} + \dfrac{1}{12}\ln 3$

4. $I = \dfrac{1}{12} + \dfrac{1}{12}\ln 3$ **correct**

5. $I = \dfrac{1}{12} + \dfrac{1}{6}\ln 3$

Explanation:

After integration by parts,

$$\begin{aligned} \int_6^{12} \frac{\ln t}{t^2}\, dt &= \Big[-\frac{\ln t}{t}\Big]_6^{12} + \int_6^{12} \frac{1}{t^2}\, dt \\ &= \Big[-\frac{\ln t}{t} - \frac{1}{t}\Big]_6^{12} \\ &= \frac{1}{12} + \Big(\frac{\ln 6}{6} - \frac{\ln 12}{12}\Big). \end{aligned}$$

This last expression can be simplified using the properties of logs. For

$$\ln 12 = \ln(2\cdot 6) = \ln 2 + \ln 6,$$

so that

$$\begin{aligned} \frac{\ln 6}{6} - \frac{\ln 12}{12} &= \Big(\frac{1}{6} - \frac{1}{12}\Big)\ln 6 - \frac{\ln 2}{12} \\ &= \frac{1}{12}(\ln 6 - \ln 2) \\ &= \frac{1}{12}\ln 3. \end{aligned}$$

Thus
$$\boxed{I = \frac{1}{12} + \frac{1}{12}\ln 3.}$$

keywords: integration by parts, log function

007 (part 1 of 1) 10 points

Determine if the improper integral

$$I = \int_4^\infty 4xe^{-4x^2}\,dx$$

converges, and if it does, find its value.

1. $I = e^{-64}$

2. $I = 4e^{-64}$

3. $I = \frac{1}{2}e^{64}$

4. I does not converge

5. $I = e^{64}$

6. $I = \frac{1}{2}e^{-64}$ **correct**

Explanation:

The integral

$$I = \int_4^\infty 4xe^{-4x^2}\,dx$$

is improper because of the infinite range of integration. To overcome this we restrict to a finite interval of integration and consider the limit

$$I = \lim_{t\to\infty} I_t, \quad I_t = \int_4^t 4xe^{-4x^2}\,dx.$$

To evaluate I_t we use the substitution $u = x^2$. For then

$$I_t = 2\int_{16}^{t^2} e^{-4u}\,du = -\frac{1}{2}\Big[e^{-4u}\Big]_{16}^{t^2}$$
$$= \frac{1}{2}\Big(e^{-64} - e^{-4t^2}\Big).$$

But,

$$\lim_{x\to\infty} x^m e^{-ax^2} = 0,$$

for any integer $m > 0$ and any $a > 0$, so

$$\lim_{t\to\infty} e^{-4t^2} = 0.$$

Consequently, I converges and

$$\boxed{I = \frac{1}{2}e^{-64}}.$$

keywords: improper integral, limit, exponential function

008 (part 1 of 1) 10 points

Determine if the improper integral

$$I = \int_0^1 4\ln 6x\,dx$$

converges, and if it does, find its value.

1. $I = 4(\ln 6 + 1)$

2. $I = 4(\ln 6 - 1)$ **correct**

3. $I = 6(\ln 4 - 1)$

4. $I = 4\ln 6 - 6$

5. $I = 6\ln 4 + 4$

6. I does not converge

7. $I = 6(\ln 4 + 1)$

Explanation:

Since $\ln 6x \to -\infty$ as $x \to 0$, the graph of $4\ln 6x$ has a vertical asymptote at $x = 0$. It is this that makes I an improper integral. So we set

$$I = \lim_{t\to 0+} \int_t^1 4\ln 6x\,dx$$

and check if the limit exists.

Now, after integration by parts,

$$\int_t^1 4\ln 6x\,dx = \Big[\,4x\ln 6x\,\Big]_t^1 - \int_t^1 4\,dx\,.$$

Thus

$$\int_t^1 4\ln 6x\,dx = 4\ln 6 - 4 - 4t\ln\,6t + 4t\,.$$

It is when investigating the limit of this expression as $t \to 0+$ that L'Hospital's Rule is needed. For

$$\lim_{t\to 0+} 4t\ln 6t = \lim_{t\to 0+} \frac{4\ln 6t}{(1/t)}\,,$$

so by L'Hospital's Rule

$$\lim_{t\to 0+} \frac{4\ln 6t}{(1/t)} = \lim_{t\to 0+} \frac{4/t}{-(1/t^2)} = 0\,.$$

Hence

$$\lim_{t\to 0+} \int_t^1 4\ln 6x\,dx$$

$$= 4\ln 6 - 4 + \lim_{t\to 0+} 4t = 4\ln 6 - 4\,.$$

Consequently,

$$\boxed{I = 4\ln 6 - 4 = 4(\ln 6 - 1)}\,.$$

---

keywords: improper integral, unbounded function, finite interval, L'Hospital, log function

---

009 (part 1 of 1) 10 points

Determine f_y when

$$f(x,\,y) = \sin(4x-y) - y\cos(4x-y)\,.$$

1. $f_y = -y\sin(4x-y)$

2. $f_y = 2\cos(4x-y) + y\sin(4x-y)$

3. $f_y = y\cos(4x-y)$

4. $f_y = 2\sin(4x-y) - y\cos(4x-y)$

5. $f_y = -2\cos(4x-y) - y\sin(4x-y)$ **correct**

6. $f_y = -2\sin(4x-y) + y\cos(4x-y)$

7. $f_y = y\sin(4x-y)$

8. $f_y = -y\cos(4x-y)$

Explanation:

From the Product Rule we see that

$$f_y = -\cos(4x-y) - \cos(4x-y) - y\sin(4x-y)\,.$$

Consequently,

$$\boxed{f_y = -2\cos(4x-y) - y\sin(4x-y)}\,.$$

---

keywords: partial derivative, first order partial derivative, trig function,

---

010 (part 1 of 1) 10 points

Find the volume of the solid under the graph of

$$f(x,y) = 3 + 9x^2 + 4y$$

and above the rectangle

$$A = \Big\{(x,y) : 1 \le x \le 2\,,\ \ 0 \le y \le 2\Big\}\,.$$

1. volume = 61 cu.units

2. volume = 54 cu.units

3. volume = 55 cu.units

4. volume = 56 cu.units **correct**

5. volume = 63 cu.units

Explanation:

The volume is given by the double integral

$$V = \int_0^2 \int_1^2 (3 + 9x^2 + 4y)\,dxdy$$

of $f(x, y)$ over the rectangular region A. Integrating each term separately, we see that

$$V = \int_0^2 \int_1^2 3\, dxdy + \int_0^2 \int_1^2 9x^2\, dxdy$$
$$+ \int_0^2 \int_1^2 4y\, dxdy = 6 + 42 + 8.$$

Consequently,

$$\boxed{\text{volume} \;=\; 56\,\text{cu.units}}.$$

keywords:

---

011 (part 1 of 1) 10 points

Evaluate the double integral

$$I = \int\int_A (4x+3)\, dxdy$$

when A is the bounded region enclosed by $y = x$ and $y = x^2$.

1. $I = \frac{2}{3}$

2. $I = \frac{7}{6}$

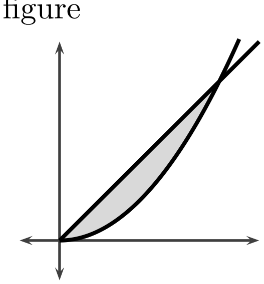
3. $I = \frac{1}{2}$

4. $I = 1$

5. $I = \frac{5}{6}$ **correct**

Explanation:

The area of integration A is the shaded region in the figure



To determine the limits of integration, therefore, we have first to find the points of intersection of the line $y = x$ and the parabola $y = x^2$. These occur when $x^2 = x$, *i.e.*, when $x = 0$ and $x = 1$. Thus the double integral can be written as a repeated integral

$$I = \int_0^1 \left[\int_{x^2}^x (4x+3) dy\right] dx,$$

integrating first with respect to y. After integration this inner integral becomes

$$\Big[(4x+3)y\Big]_{x^2}^x = (4x+3)(x-x^2)$$
$$= 3x + x^2 - 4x^3.$$

Thus

$$I = \int_0^1 (3x + x^2 - 4x^3)\, dx$$
$$= \left[\frac{3}{2}x^2 + \frac{1}{3}x^3 - x^4\right]_0^1.$$

Consequently,

$$\boxed{I = \frac{5}{6}}.$$

keywords:

---

012 (part 1 of 1) 10 points

Reverse the order of integration in the integral

$$I = \int_4^{\sqrt{24}} \Big(\int_{x^2/6}^4 f(x, y)\, dy\Big) dx,$$

but make no attempt to evaluate either integral.

1. $I = \int_{y^2/6}^4 \Big(\int_4^{\sqrt{24}} f(x, y)\, dx\Big) dy$

2. $I = \int_{\frac{8}{3}}^4 \Big(\int_{4y}^{\sqrt{6}} f(x, y)\, dx\Big) dy$

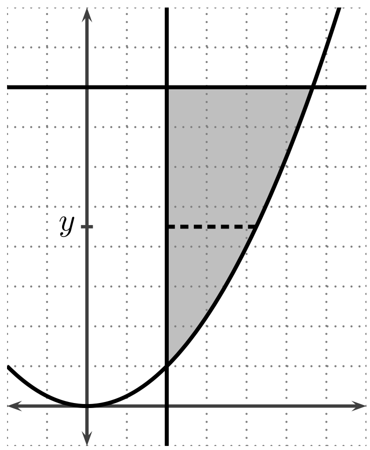
3. $I = \int_{\frac{8}{3}}^4 \Big(\int_4^{\sqrt{6y}} f(x, y)\, dx\Big) dy$ **correct**

4. $I = \int_4^6 \Big(\int_{\sqrt{6}}^{4y} f(x, y)\, dx \Big) dy$

5. $I = \int_{\frac{8}{3}}^4 \Big(\int_{\sqrt{6y}}^{4} f(x, y)\, dx \Big) dy$

Explanation:

The region of integration is similar to the shaded region in the figure



(not drawn to scale). This shaded region is enclosed by the graphs of

$$6y = x^2, \qquad y = 4, \qquad x = 4.$$

To change the order of integration, first fix y. Then, as indicated by the dashed line , x varies from 4 to $\sqrt{6y}$. To cover the region of integration, therefore, y must vary from $\frac{8}{3}$ to 4. Hence, after changing the order of integration,

$$\boxed{I = \int_{\frac{8}{3}}^4 \Big(\int_4^{\sqrt{6y}} f(x, y)\, dx \Big) dy}.$$

keywords:

013 (part 1 of 1) 10 points

Determine if the sequence $\{a_n\}$ converges, and if it does, find its limit when

$$a_n = \frac{3n + (-1)^n}{6n + 1}.$$

1. converges with limit $= \frac{1}{2}$ **correct**

2. converges with limit $= \frac{2}{3}$

3. converges with limit $= \frac{1}{3}$

4. converges with limit $= \frac{3}{7}$

5. sequence does not converge

Explanation:

After division by n we see that

$$a_n = \frac{3 + \dfrac{(-1)^n}{n}}{6 + \dfrac{1}{n}}.$$

But

$$\frac{(-1)^n}{n}, \quad \frac{1}{n} \longrightarrow 0$$

as $n \to \infty$, so $a_n \to \frac{1}{2}$ as $n \to \infty$. Consequently, the sequence converges and has

$$\boxed{\text{limit} = \frac{1}{2}}.$$

keywords: sequence, convergence, properties of limits

014 (part 1 of 1) 10 points

Determine if the sequence $\{a_n\}$ converges, and if it does, find its limit when

$$a_n = \int_n^{7n} \frac{1}{2x + 5}\, dx.$$

1. limit $= \frac{1}{2} \ln \frac{7}{2}$

2. the sequence diverges

3. limit $= \frac{1}{2} \ln \frac{1}{7}$

4. limit $= \ln 7$

5. limit $= \dfrac{1}{2} \ln 7$ **correct**

Explanation:

After integration,

$$a_n = \left[\frac{1}{2} \ln(2x+5) \right]_n^{7n} = \frac{1}{2} \ln\left(\frac{14n+5}{2n+5} \right).$$

Now

$$\frac{14n+5}{2n+5} = \frac{14 + \frac{5}{n}}{2 + \frac{5}{n}} \longrightarrow 7$$

as $n \to \infty$. But $\ln x$ is continuous as a function of x. We thus see that $\{a_n\}$ converges as $n \to \infty$ and has

$$\boxed{\text{limit} = \frac{1}{2} \ln 7}.$$

keywords: sequence, limit, log

---

015 (part 1 of 1) 10 points

If the n^{th} partial sum of an infinite series is

$$S_n = \frac{n^2 - 2}{4n^2 + 1},$$

what is the sum of the series?

1. sum $= \dfrac{5}{16}$

2. sum $= \dfrac{1}{4}$ **correct**

3. sum $= \dfrac{1}{8}$

4. sum $= \dfrac{3}{8}$

5. sum $= \dfrac{3}{16}$

Explanation:

By definition

$$\text{sum} = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(\frac{n^2-2}{4n^2+1} \right).$$

Thus

$$\boxed{\text{sum} = \frac{1}{4}}.$$

keywords: partial sum, definition of series

---

016 (part 1 of 1) 10 points

Determine if the series

$$\sum_{n=1}^{\infty} \frac{2+4^n}{5^n}$$

converges or diverges, and if it converges, find its sum.

1. converges with sum $= \dfrac{15}{4}$

2. converges with sum $= \dfrac{19}{4}$

3. series diverges

4. converges with sum $= \dfrac{9}{2}$ **correct**

5. converges with sum $= 4$

6. converges with sum $= \dfrac{17}{4}$

Explanation:

An infinite geometric series $\sum_{n=1}^{\infty} a\, r^{n-1}$

(i) converges when $|r| < 1$ and has

$$\text{sum} = \frac{a}{1-r},$$

while it

(ii) diverges when $|r| \geq 1$.

Now

$$\sum_{n=1}^{\infty} \frac{2}{5^n} = \sum_{n=1}^{\infty} \frac{2}{5}\left(\frac{1}{5}\right)^{n-1}$$

is a geometric series with $a = r = \frac{1}{5} < 1$. Thus it converges with

$$\text{sum} = \frac{1}{2},$$

while

$$\sum_{n=1}^{\infty} \frac{4^n}{5^n} = \sum_{n=1}^{\infty} \frac{4}{5}\left(\frac{4}{5}\right)^{n-1}$$

is a geometric series with $a = r = \frac{4}{5} < 1$. Thus it too converges, and it has

$$\text{sum} = 4.$$

Consequently, being the sum of two convergent series, the given series

$$\boxed{\text{converges with sum} = \frac{1}{2} + 4 = \frac{9}{2}}.$$

---

keywords: geometric series