M 408L Integral Calculus **Prepared by Chris Mirabito**
Test #2 Review Sheet ("Cookbook") **March 20, 2007**

Test #2 will be held on Tuesday, March 27, 2007 at 7:00pm in GSB 2.124 (**THIS IS A DIFFERENT LOCATION THAN EXAM 1**). The test will over §8.1, §8.2, §8.3, §8.4, §8.5, §8.7, §8.8, §15.3, §16.1, §16.2, and §16.3 in the course textbook.[1] You **cannot** use a calculator (or any other electronic devices), books, or notes in any form. The test will be 2 hours long and will be worth 20% of your final course grade. On the test will be up to 20 multiple-choice questions similar to what you have found on Homeworks #5 through #9. Unlike the homework problems, you will NOT lose points for guessing.

Please bring your UT ID cards to the test. We will be checking them: your cooperation with the University policy on cheating is expected. Also, remember to bring a #2 pencil and eraser to the test, as none will be provided. All other testing materials will be provided.

This review sheet is intended to help you prepare for the test. It states in a nutshell pretty much everything that we have covered in the course since Test #1, but *this list is not exhaustive*. Please be sure you are familiar with all concepts listed here. If you are having trouble, please stop by Dr. Fouli's or your TA's office hours. Do not wait until the last minute to ask for help. And remember, material needed for Test #1 could very well be used again on this test. . . that means volumes, exponentials, etc.

Darice Chang will hold a review session Friday, March 23rd, 2007 from 2:00pm until 4:00pm in BUR 112. I highly recommend attending this, since it will help answer any last-minute questions you may have.

There is a practice test for you attached at the end. It is in a format similar to the actual test. You should take this test in a quiet area without books, notes, and calculators. Hopefully it will help you study.

♿ Students with legitimate, documented learning disabilities or special needs should consult the "First Day Handout" located at

`http://www.ma.utexas.edu/dev/math/Courses/M408L/handout.html`

for information regarding testing dates, times, and locations (which will be different from that of the regular time). Students who cannot take the test at the regularly scheduled time due to conflicts with other classes, exams, or documented illnesses should also consult this page for make-up test information. In all cases, make sure Dr. Fouli knows about your situation well in advance.

§8.1 Integration by Parts

- Memorize this formula: $\int u\,dv = uv - \int v\,du$
- For definite integrals: $\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$
- It helps to make a little table for u, du, v, and dv. After filling in this little table, all you have to do is plug n' chug.
- How to choose u: **LIATE**

  L ogarithmic functions

  I nverse Trigonometric functions

[1] If you have another version or a previous edition of the textbook, the sections to you need to know may be slightly different. If you are not sure which sections you should study, please ask me.

A lgebraic (i.e. polynomials, radicals, etc.)

T rigonometric functions

E xponentials

- Your choice of u and dv should make the resulting integral (on the right hand side) easier to solve, not harder. If you find that your resulting integral is harder to solve than the original, you probably made the wrong choice of u and dv. Try a different choice instead.
- It is a good idea to select u such that it's easy to take its derivative and dv such that it is easy to integrate.

§8.2 Trigonometric Integrals

- This is a tricky section that will require some cleverness on your part. Study this section very carefully, and proceed with caution. *You need to know trigonometric identities in order to do well in this section.*
- Study the boxes on pages 520 and 522, which describe some basic strategy for integrating $\int \sin^m x \cos^n x\, dx$ and $\int \tan^m x \sec^n x\, dx$.
- In general, you shouldn't try to use u-substitution until you simplify the integrand using 1 or more trigonometric identities.
- If you have $\int \sin^m x \cos^n x\, dx$:
  1. If m is odd, split off a copy of $\sin x$, let $\sin^2 x = 1 - \cos^2 x$, then let $u = \cos x$.
  2. If n is odd, split off a copy of $\cos x$, let $\cos^2 x = 1 - \sin^2 x$, then let $u = \sin x$.
  3. If both powers are odd, pick your poison!
  4. If both powers are even, let $\sin^2 x = \dfrac{1-\cos 2x}{2}$ and $\cos^2 x = \dfrac{1+\cos 2x}{2}$.
- If you have $\int \tan^m x \sec^n x\, dx$:
  1. If m is odd, split off a copy of $\sec x \tan x$, let $\tan^2 x = \sec^2 x - 1$, then let $u = \sec x$.
  2. If n is even, let $\sec^2 x = 1 + \tan^2 x$, then let $u = \tan x$.
  3. If m is odd **and** n is even, pick your poison!
  4. If m is even **and** n is odd, try using the identity $\tan^2 x = \sec^2 x - 1$.
- Remember that $\int \tan x\, dx = \ln|\sec x| + C$ and $\int \sec x\, dx = \ln|\sec x + \tan x| + C$.
- If you get stuck, and nothing else seems to be working, don't be afraid to multiply the integrand by factors like $\dfrac{\sec x}{\sec x}$ or $\dfrac{\sec x + \tan x}{\sec x + \tan x}$. You will need to do this when integrating $\sec x$ or $\csc x$, for example.

§8.3 Trigonometric Substitution

- *You need to know trigonometric identities to do well in this section, too.*
- The basic idea behind this:

If you see...	Substitute...	and simplify using...
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

- Remember that you can use this when you see quantities like $(x^2+a^2)^{3/2}$ or $(x^2+a^2)^{2/3}$ too.
- Remember also to draw a triangle to figure out the values for the other trigonometric functions *after you have integrated*.
- Expression not in one of the forms above? Try completing the square and factoring.

§8.4 Integration of Rational Functions by Partial Fractions

- Before trying to use partial fractions, try to simplify: if $\deg(numerator) \geq \deg(denominator)$, you must divide using polynomial long division.
- Factor all terms in the denominator, if possible.
- The 4 cases to consider, and what to do:
  1. *Distinct Linear Factors*: Split into form $\dfrac{A}{ax+b}+\dfrac{B}{cx+d}+\cdots$
  2. *Repeated Linear Factors*: Split into form $\dfrac{A}{ax+b}+\dfrac{B}{(ax+b)^2}+\cdots$
  3. *Distinct Quadratic Factors*: Split into form $\dfrac{Ax+B}{ax^2+bx+c}+\dfrac{Cx+D}{dx^2+fx+g}+\cdots$
  4. *Repeated Quadratic Factors*: Split into form $\dfrac{Ax+B}{ax^2+bx+c}+\dfrac{Cx+C}{(ax^2+bx+c)^2}+\cdots$
- NOTE: In cases 3 and 4, the quadratic factors **must be irreducible**. This will not work if the factors are reducible.
- Often you'll have mixtures of these cases. Just remember to treat each factor separately. For example,

$$\frac{1}{(4x+5)^2(5x^2+6x+7)^2}=\frac{A}{4x+5}+\frac{B}{(4x+5)^2}+\frac{Cx+D}{5x^2+6x+7}+\frac{Ex+F}{(5x^2+6x+7)^2}.$$

§8.5 Strategy for Integration

- It is important to *read the book and/or your notes* here. It gives details on what to look for in integrands.
- The basic strategy is:
  1. Simplify the integrand, if possible.
  2. Try using u-substitution.
  3. Look at the integrand. Do you have trigonometric functions (like in §8.2), rational functions (like in §8.4), or a product to 2 things (with 1 easy to differentiate and the other easy to integrate…do integration by parts here), or things like $(x^2 \pm a^2)^n$ (do trigonometric substitution here)?
  4. Try again (i.e. you'll have to be a little more crafty).

§8.7 Approximate Integration

- We have 5 methods of approximating an integral: the **Left Endpoint Rule**, the **Right Endpoint Rule**, the **Midpoint Rule**, the **Trapezoid Rule**, and **Simpson's Rule**. Some are more accurate than others (do you remember which ones I said were best?)
- **Trapezoid Rule:** $\dfrac{\Delta x}{2}[f(x_0)+2f(x_1)+\cdots+2f(x_{n-1})+f(x_n)]$.

- **Simpson's Rule:** $\frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)]$. Remember that n must be even here. Make sure you know the order of the coefficients! (Draw the mountains to help you.)
- You do NOT need to know the formulas for the error estimates for the Trapezoid Rule and Simpson's Rule.
- Many people asked me whether they would need to evaluate the approximation to a decimal. No need to freak out here. The choices will be expressions for the approximation, not decimals (unless they're really easy to evaluate).

§8.8 Improper Integrals

- How to set them up:
  1. $\int_a^\infty f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx$
  2. $\int_{-\infty}^b f(x)\,dx = \lim_{s\to-\infty}\int_s^b f(x)\,dx$
  3. $\int_{-\infty}^\infty f(x)\,dx = \lim_{s\to-\infty}\int_s^p f(x)\,dx + \lim_{t\to\infty}\int_p^t f(x)\,dx$
- In the setup above, pick p to be something convenient, like 0.
- If $f(x)$ has a vertical asymptote (check for this!), split the integral there, and then take left and right hand limits.
- If you split an integral and one portion diverges, **stop**. The integral diverges.
- Remember to carry the limits all the way through. Not doing this will confuse you.

§15.3 Partial Derivatives

- Do not let notation confuse you: if $z = f(x,y)$ then

  $$f_x(x,y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x,y) = \frac{\partial z}{\partial x} = f_1 = D_1 f = D_x f$$

  and

  $$f_y(x,y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} f(x,y) = \frac{\partial z}{\partial y} = f_2 = D_2 f = D_y f.$$

- To find $\frac{\partial f}{\partial x}$, differentiate with respect to x and *treat y as a constant*. And to find $\frac{\partial f}{\partial y}$, differentiate with respect to y and hold x constant.
- What if you have $u(x,y,z,w,v,\theta,\alpha,\beta,\gamma,\delta,\varepsilon)$ and you wanted u_x? Differentiate with respect to x and hold *all other variables* constant.
- Remember the physical meaning of a partial derivative: $f_x(a,b)$ is the slope of f in the x-direction at (a,b), and similarly for y.
- Clairaut's Theorem: $f_{xy} = f_{yx}$ as long as f and all its partial derivatives are continuous. In other words, if f, f_x, f_y, f_{xy} and f_{yx} are continuous, then you can switch the order in which you differentiate and you'll get the same answer!

§16.1 Double Integrals over Rectangles

- Remember that the Fundamental Theorem of Calculus (both parts) holds for functions with 2 or more variables too. All the properties we learned in §5.1 and §5.2 still work.
- You can also use the same methods for approximating a double integral as before: Left Endpoints, Right Endpoints, etc. This time though, it's possible to use Left Endpoints along x and Right Endpoints along y, etc.!

§16.2 Iterated Integrals

- The expression $\int_a^b \left[\int_c^d f(x,y)\,dx\right] dy$ is an iterated integral. To solve these, integrate with respect to one variable at a time, inside out.
- Fubini's Theorem: If $f(x,y)$ is continuous then $\int_a^b \int_c^d f(x,y)\,dx\,dy = \int_c^d \int_a^b f(x,y)\,dy\,dx$. In other words, as long as f is continuous, you can change the order of integration. But remember, switching dx and dy means you must change the limits of integration too!
- As a reminder, as before, you can pull constants out of both integrals. But if you're integrating with respect to, say, x first and your integrand is something like $yf(x)$, you can only pull the y out of one of the integrals, not both!

§16.3 Double Integrals over General Regions

- When you change the order of integration, you must change your limits so that the ones on the inner integral are in terms of the other variable.
- It is helpful to draw the region that you are integrating over. If you choose to integrate with respect to x first, draw a horizontal line *anywhere* through the region. If you choose to integrate with respect to y first, draw a vertical line anywhere through the region.
- Your limits on the inner integral will be functions. The lower limit is where you *enter* the region, and the upper limit is where you *leave* the region.
- The limits on the outer integral will always be numbers. *You should never have variables in your limits of integration on the outer integral. Ever. Ever!*

GOOD LUCK!! If you have any last-minute questions, email me or Dr. Fouli.

M 408L Integral Calculus
Test #2 Practice Problems **March 20, 2007**

This practice test is intended to help you study for Test #2. It includes 17 multiple-choice questions which may be similar to what you may be asked to do on the actual test. You should complete these questions on your own, in a quiet area, without books, notes, or any electronic devices. You should give yourself 2 hours to do this practice test so you can learn to pace yourself (about 6 minutes per problem). Hopefully, these questions will be **harder** than the ones that will be on the test.

Use a separate sheet of paper for your work. A page to mark your answers and a solution page are located on the last page.

1. $\int \tan^5 x\, dx =$

   (a) $x\tan^4 x - 5x\tan^4 x\sec^2 x + C$

   (b) $\dfrac{\tan^6 x}{6} + C$

   (c) 0

   (d) $\dfrac{1}{4}\tan^4 x - \dfrac{1}{2}\tan^2 x + \ln|\sec x| + C$

   (e) $\dfrac{\tan^3 x}{3} - \tan x + \ln|\sec x| + C$

2. $\int \sin^4 x\cos^4 x\, dx =$

   (a) $\dfrac{5}{8}x + \dfrac{\sin 2x}{16} + C$

   (b) $\dfrac{\sin 2x}{2} + C$

   (c) $\dfrac{\sin^8 x}{2} + C$

   (d) $\dfrac{\sin^4 2x}{16} + \dfrac{\sin^4 4x}{64} + C$

   (e) $\dfrac{3}{128}x - \dfrac{\sin 4x}{128} + \dfrac{\sin 8x}{1024} + C$

3. $\int 2\cos(\ln x)\, dx =$

   (a) $2x\{\sin(\ln x) + \cos(\ln x)\} + C$

   (b) $2x\{\sin(\ln x) - \cos(\ln x)\} + C$

   (c) $x\{\sin(\ln x) - \cos(\ln x)\} + C$

   (d) $x\{\sin(\ln x) + \cos(\ln x)\} + C$

   (e) $x\{\cos(\ln x) - \sin(\ln x)\} + C$

4. $\int_0^{1/2} 8\sin^{-1} x\, dx =$

   (a) $2\pi + 4(2 - \sqrt{3})$

   (b) $2\pi - 4(2 + \sqrt{3})$

(c) $\pi+4(2-\sqrt{3})$

(d) $\dfrac{2\pi}{3}+4(2+\sqrt{3})$

(e) $\dfrac{2\pi}{3}-4(2-\sqrt{3})$

5. $\displaystyle\int 3\sin^2 x\cos^3 x\,dx =$

   (a) Divergent

   (b) $\sin^3 x-\dfrac{3}{5}\sin^5 x+C$

   (c) $\sin^3 x+\dfrac{3}{5}\sin^5 x+C$

   (d) $\dfrac{3}{5}\cos^3 x-\sin^5 x+C$

   (e) $-\dfrac{3}{5}\sin^3 x-\cos^5 x+C$

6. Which of the following integrals on the interval $\left[0,\dfrac{\pi}{4}\right]$ has the greatest value?

   (a) $\displaystyle\int_0^{\pi/4}\sin t\,dt$

   (b) $\displaystyle\int_0^{\pi/4}\cos t\,dt$

   (c) $\displaystyle\int_0^{\pi/4}\cos^2 t\,dt$

   (d) $\displaystyle\int_0^{\pi/4}\cos 2t\,dt$

   (e) $\displaystyle\int_0^{\pi/4}\sin t\cos t\,dt$

7. $\displaystyle\int\frac{3}{\sqrt{5+4x-x^2}}\,dx =$

   (a) $3\tan^{-1}\left(\dfrac{x-2}{3}\right)+C$

   (b) $3\sin^{-1}\left(\dfrac{x-2}{3}\right)+C$

   (c) $3\tan^{-1}\left(\dfrac{x-3}{2}\right)+C$

   (d) $\sin^{-1}\left(\dfrac{x+2}{3}\right)+C$

   (e) $\tan^{-1}\left(\dfrac{x+3}{2}\right)+C$

8. Find a solution to the first-order differential equation

$$\frac{dy}{dx}=\frac{2}{(x-1)(x+3)^2}$$

   with an unrestricted initial condition.

(a) $y(x) = -\frac{1}{8}\ln|x-1| + \frac{1}{8}\ln|x+3| - \frac{1}{2(x+3)} + C$

(b) $y(x) = -\frac{1}{8}\ln|x-1| - \frac{1}{8}\ln|x+3| + \frac{1}{2(x+3)} + C$

(c) $y(x) = -\frac{1}{8}\ln|x-1| + \frac{1}{8}\ln|x+3| + \frac{1}{2(x+3)} + C$

(d) $y(x) = \frac{1}{8}\ln|x-1| - \frac{1}{8}\ln|x+3| + \frac{1}{2(x+3)} + C$

(e) $y(x) = \frac{1}{2}\ln|x-1| - \frac{1}{8}\ln|x+3| + \frac{1}{8(x+3)} + C$

9. The Dodge Viper SRT10 can go from 0 to 60 mph in less than 4 seconds. Suppose velocity was measured according to the following table:

t (s)	$v(t)$ (mph)
0	0
0.5	0.125
1	1
1.5	3.375
2	8
2.5	15.625
3	27
3.5	42.875
4	64

Use Simpson's Rule to determine the total distance traveled by the Viper (i.e. the "lengths" it will go to try to get you to spend $85,745.00 on a car!).

(a) $\frac{1}{21600}[4(0.125)+2(1)+4(3.375)+2(8)+4(15.625)+2(27)+4(42.875)+64]$ miles

(b) $\frac{1}{14400}[4(0.125)+2(1)+4(3.375)+2(8)+4(15.625)+2(27)+4(42.875)+64]$ miles

(c) $\frac{1}{21600}[2(0.125)+4(1)+2(3.375)+4(8)+2(15.625)+4(27)+2(42.875)+64]$ miles

(d) $\frac{1}{14400}[2(0.125)+2(1)+2(3.375)+2(8)+2(15.625)+2(27)+2(42.875)+64]$ miles

(e) $\frac{1}{7200}[0.125+1+3.375+8+15.625+27+42.875+64]$ miles

10. $\int_0^3 \frac{dx}{(1-x)^2} =$

(a) $-\frac{3}{2}$

(b) $-\frac{1}{2}$

(c) $\frac{1}{2}$

(d) $\frac{3}{2}$

(e) Divergent

11. $\int_0^1 \frac{\ln 4x}{\sqrt{x}}\,dx =$

(a) Divergent

(b) $\ln 4 - 2$

(c) $2(\ln 4 - 2)$

(d) $4(\ln 4 + 1)$

(e) $4(\ln 4 - 1)$

12. A key measure of a nation's economic strength is the rate of inflation. In 2005, the U.S. inflation rate was measured at 4% per year, meaning \$1.04 today could buy \$1 worth of goods in 2005. Now, typically the inflation rate is measured monthly, but suppose that it is possible to measure the inflation rate continuously. Suppose also that in the long-term, the Fed manages to stabilize the inflation rate so that the inflation rate $I(t) = 0.04e^{-t}\cos t$, where t is measured in years and $t = 0$ corresponds to 2005. Assuming that $I(t)$ continues on this trend forever, which best describes the long-term U.S. economic trend?

(a) \$1.02 will buy \$1 worth of 2005 goods.

(b) \$0.98 will buy \$1 worth of 2005 goods.

(c) \$1 will buy \$1 worth of 2005 goods.

(d) Meltdown for the U.S. monetary system!

(e) Cannot tell from the given data.

13. If $f(x,y,z) = \sin xy \cos yz$, then $f_{yz}(x,y,0) =$

(a) π

(b) $\frac{\pi}{2}$

(c) -1

(d) 1

(e) 0

14. If $z = x\sec y$, then $\frac{\partial^2 z}{\partial x \partial y} =$

(a) 0

(b) $\csc y$

(c) $\sec^2 y$

(d) $\sec y \tan y$

(e) $\tan y - 1$

15. Which of the following integrals on $[0,1]\times[0,1]$ has the smallest value?

(a) $\int_0^1 \int_0^1 \ln x \ln y\,dx\,dy$

(b) $\int_0^1 \int_0^1 e^x e^y\,dx\,dy$

(c) $\int_0^1 \int_0^1 xy\,dx\,dy$

(d) $\int_0^1 \int_0^1 \sqrt{x}\sqrt{y}\,dx\,dy$

(e) $\int_0^1 \int_0^1 \frac{1}{x}\frac{1}{y}\, dx\, dy$

16. $\int_0^3 \int_{-2}^2 \frac{3xy^2}{16+x^2}\, dy\, dx =$

(a) $8\ln\left(\frac{16}{25}\right)$

(b) $4\ln\left(\frac{25}{16}\right)$

(c) $4\ln\left(\frac{16}{25}\right)$

(d) $4\ln\left(\frac{25}{32}\right)$

(e) $8\ln\left(\frac{25}{26}\right)$

17. If D is the triangle with vertices at $(0,2)$, $(1,1)$, and $(3,2)$, then $\iint_D y^3\, dA =$

(a) $-\frac{3}{20}$

(b) $\frac{3}{20}$

(c) $\frac{147}{20}$

(d) $\frac{20}{147}$

(e) $\frac{20}{3}$

ANSWER SHEET

1. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
8. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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11. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
12. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
13. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
14. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
15. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
16. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
17. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

ANSWERS

1. D
2. E
3. D
4. E
5. B
6. B
7. B
8. D
9. A
10. E
11. C
12. A
13. E
14. D
15. C
16. E
17. C