M 408C
September 13, 2010

1. Find a function f and a number a so that
$$f'(a) = \lim_{x \to 3\pi} \frac{\cos x + 1}{x - 3\pi}.$$

2. The fuel consumption (in gal/hr) of a car traveling at a speed of v miles per hour is $c = f(v)$.
   (a) What is the meaning of the derivative $f'(v)$? What are its units?
   (b) Write a sentence to explain the meaning of $f'(20) = -0.05$.

3. Find the derivative of $f(x) = x + \sqrt{x}$ using the definition of derivative (you do not need to use the definition of limit, though). Does the answer match with the formula given by the Power Rule? What is the domain of f? What is the domain of f'?

4. Compute $\lim_{x \to 1} \frac{x^{50}-1}{x-1}$.

5. Find the derivatives of the following:
$$f(x) = \frac{1}{x\sqrt[3]{x}} \qquad g(x) = \frac{ax+b}{cx+d} \qquad h(t) = \sqrt{t}(t^3 - \sqrt{t})$$
$$s(t) = \tan(t)(t^4 - 4t) \quad P(x) = A + \frac{B}{x} + \frac{C}{x^2} \qquad f(z) = \frac{1}{\sqrt[5]{2z-1}}$$

6. Find an equation of the tangent line to $f(x) = \sin(\sin x)$ at $(\pi, f(\pi))$.

7. Find an equation of the tangent line to $g(x) = \sec\left(\frac{\pi x}{x+3}\right)$ at $(1, g(1))$.

8. The Binomial Theorem states that for any positive integer n,
$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \ldots + \frac{n!}{(n-i)!i!}x^{n-i}y^i + \ldots + nxy^{n-1} + y^n.$$
   Use this theorem to prove the Power Rule for integer exponents: $\frac{d}{dx}(x^n) = nx^{n-1}$.