## M 408C

## September 15, 2010

1. Find the derivatives of the following:

$$
\begin{array}{ccc}
f(u)=\sqrt{2} u+\sqrt{3 u} & g(z)=\left(z^{-2}+z^{-3}\right)\left(z^{5}-2 z^{2}\right) & h(t)=\csc t(t+\cot t) \\
f(\theta)=\sin \sqrt{\theta} & g(\theta)=\sqrt{\sin \theta} & h(\theta)=\tan ^{2}(3 \theta)
\end{array}
$$

2. Find an equation of the tangent line to $y=\frac{2 x}{x+1}$ at $(1,1)$.
3. Find an equation of the tangent line and the normal line to $g(x)=(1+2 x)^{2}$ at $(1,9)$.
4. Assuming that we know the derivatives of $\sin x$ and $\cos x$, derive the derivatives of the other four trigonometric functions.
5. What is $\frac{d y}{d x}$ at $(6,2)$ when $y^{6}+x^{2}=100$ ? What about at $(6,-2)$ ?
6. Is it true that all differentiable functions are continuous? Is it true that all continuous functions are differentiable? Is it true that the derivative of a differentiable function is continuous? Give arguments or counterexamples.
7. Find $\frac{d}{d x}|x|$ using the piecewise definition of $|x|$. Find $\frac{d}{d x}|x|$ using the fact that $|x|=\sqrt{x^{2}}$. Do your answers agree? Find $\frac{d}{d x}|\cos x|$.
