

M 408C
September 8, 2010

1. Determine whether the function

$$f(x) = \begin{cases} 1, & \text{if } x = 0 \\ x^2 \sin(1/x), & \text{if } x \neq 0 \end{cases}$$

is continuous. If it isn't continuous, can you change it slightly so that it is?

2. Prove that $f(x) = 3x$ is continuous.
3. Prove that $f(x) = x^2$ is continuous at $x = 3$, then prove that f is a continuous function.
4. Compute $\lim_{h \rightarrow 0} \frac{f(4+h) - f(4)}{h}$ for the functions in the two problems above. Sketch a graph of each function and interpret this limit.
5. Compute $\lim_{x \rightarrow 4} \frac{f(x) - f(4)}{x - 4}$ for $f(x) = x^2$. Interpret this limit graphically.
6. Show that $g(x) = \frac{1}{3}(\sin x - 1)$ has a fixed point.
7. Estimate $\sqrt{1598}$ and $\sqrt[3]{8.01}$.
8. Suppose that $f(x) = a^x$, where $a > 0$. Show that the instantaneous rate of change is proportional to the height of the function.
9. (Only do this one if your group has completed the other exercises and wants something challenging to work on.) Let f be defined by

$$f(x) = \begin{cases} 0, & \text{if } x \notin \mathbb{Q} \\ \frac{1}{b}, & \text{if } x \in \mathbb{Q} \text{ (where } x = \frac{a}{b} \text{ is in simplest form)} \end{cases}$$

(Remember that \mathbb{Q} is the set of rational numbers). Is f continuous anywhere?