Name:
Class time:

M305G - Practice Exam III - November 2008

Carefully read each question, and be sure to show your work.

1. (a) Convert these angles to radians: $260°$, $-400°$

$$260° \cdot \frac{\pi}{180°} = \boxed{\frac{13\pi}{9}}$$

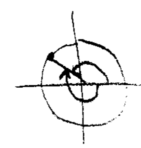
$$-400° \cdot \frac{\pi}{180°} = \boxed{-\frac{20\pi}{9}}$$

(b) Convert these angles to degrees: $-\frac{\pi}{8}$, $\frac{9\pi}{2}$

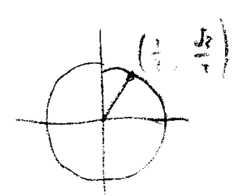
$$-\frac{\pi}{8} \cdot \frac{180°}{\pi} = -\frac{45}{2}° = \boxed{-22.5°}$$

$$\frac{9\pi}{2} \cdot \frac{180°}{\pi} = \boxed{810°}$$

(c) Find the exact value of the expression: $\cos\left(-\frac{5\pi}{4}\right) - \cos\frac{3\pi}{4} = \boxed{0}$



(d) Find the exact value of the expression: $2\sin^2 60° - 3\cos 45°$



$$2 \cdot \left(\frac{\sqrt{3}}{2}\right)^2 - 3\left(\frac{\sqrt{2}}{2}\right)$$

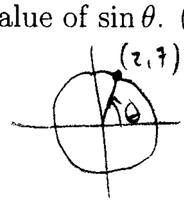
$$2 \cdot \frac{3}{4} - \frac{3\sqrt{2}}{2} = \boxed{\frac{3 - 3\sqrt{2}}{2}}$$

(e) If $f(x) = \sin x$ and $f(a) = \frac{3}{5}$, find $f(-a)$.

sin odd $\Rightarrow$ function

$$f(-a) = -f(a) = \boxed{-\frac{3}{5}}$$

(f) If the point $(2, 7)$ is on the terminal side of the angle θ in standard position, find the exact value of $\sin\theta$. (Hint: what is r?)



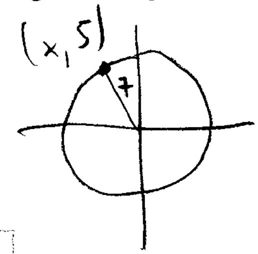
$4 + 49 = r^2$

$53 = r^2$

$$\sin\theta = \frac{y}{r} = \boxed{\frac{7}{\sqrt{53}}} = \boxed{\frac{7\sqrt{53}}{53}}$$

2. In these problems, find the value of the remaining five trigonometric functions of θ:

(a) $\sin\theta = \frac{5}{7}$, θ in quadrant II



$x^2 + 25 = 49$

$x^2 = 24$

$x = -2\sqrt{6}$

$y = 5$

$r = 7$

$\cos\theta = \frac{x}{r} = \boxed{-\frac{2\sqrt{6}}{7}}$

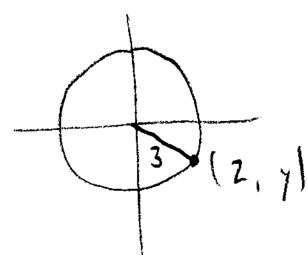
$\tan\theta = \frac{y}{x} = -\frac{5}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \boxed{-\frac{5\sqrt{6}}{12}}$

$\csc\theta = \frac{1}{\sin\theta} = \boxed{\frac{7}{5}}$

$\sec\theta = \frac{1}{\cos\theta} = -\frac{7}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \boxed{\frac{-7\sqrt{6}}{12}}$

$\cot\theta = \frac{1}{\tan\theta} = \boxed{-\frac{2\sqrt{6}}{5}}$

(b) $\cos\theta = \frac{2}{3}$, $\frac{3\pi}{2} < \theta < 2\pi$



$x = 2$

$r = 3$

$4 + y^2 = 9$

$y = -\sqrt{5}$

$\sin\theta = \frac{y}{r} = -\frac{\sqrt{5}}{3}$

$\tan\theta = \frac{y}{x} = -\frac{\sqrt{5}}{2}$

$\csc\theta = \frac{r}{y} = -\frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$

$\sec\theta = \frac{r}{x} = \frac{3}{2}$

$\cot\theta = \frac{x}{y} = -\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

3. This problem concerns the properties and graphs of sinusoidal functions.

(a) Construct a function $f(x)$ for a sinusoidal graph with the following properties:

$$A = -3, \text{ period } = \frac{2\pi}{3}, \text{ and phase shift } = -\frac{\pi}{4}$$

$$\omega = \frac{2\pi}{\frac{2\pi}{3}} = 2\pi \cdot \frac{3}{2\pi} = 3$$

$$\frac{\phi}{\omega} = -\frac{\pi}{4} \Rightarrow \phi = \omega\left(-\frac{\pi}{4}\right) = -\frac{3\pi}{4}$$

$$f(x) = A\sin(\omega x - \phi) = \boxed{-3\sin\left(3x + \frac{3\pi}{4}\right)}$$

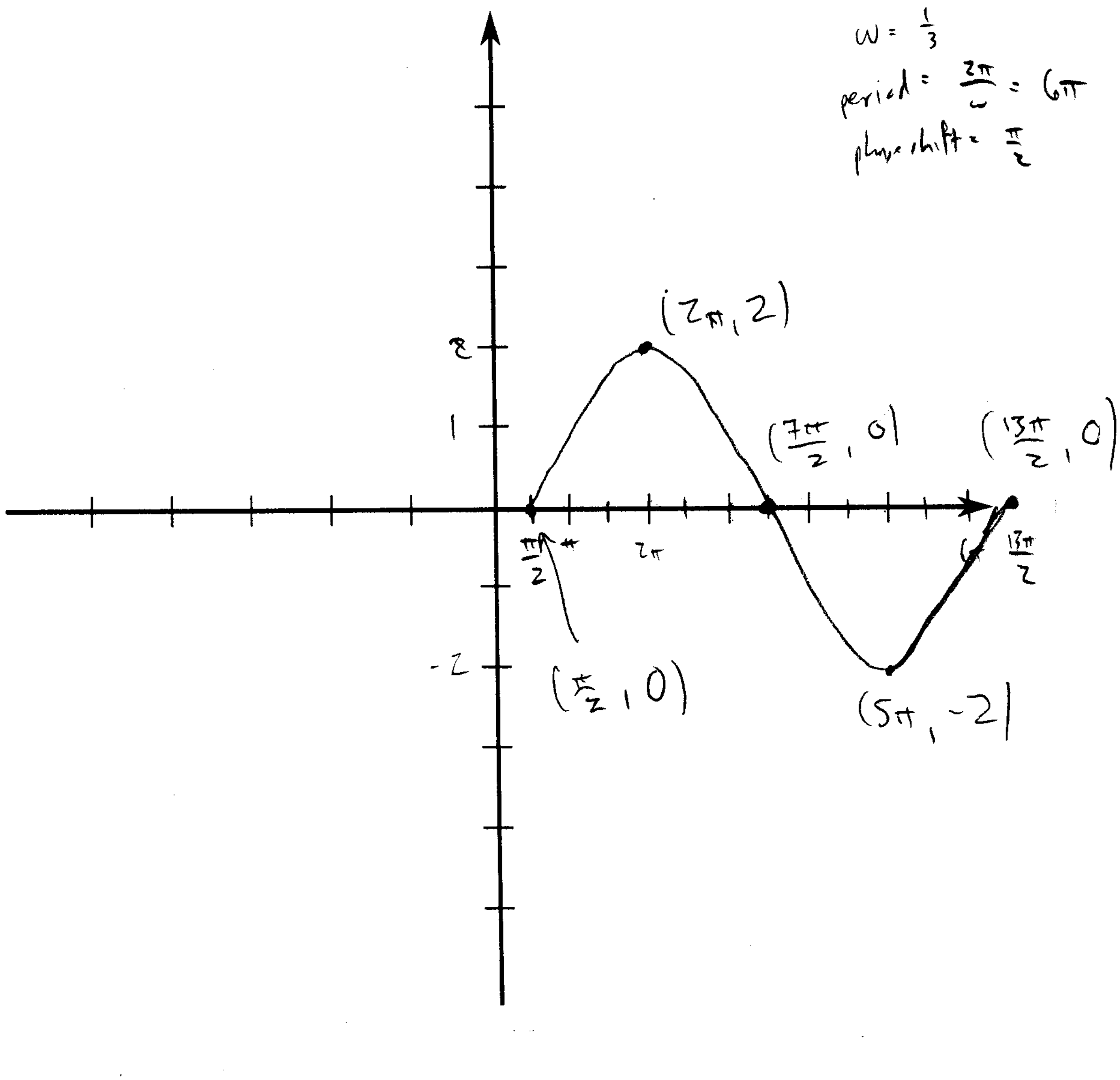
(b) Graph the function $y = 2\sin\left(\frac{x}{3} - \frac{\pi}{6}\right) = 2\sin\left(\frac{1}{3}\left(x - \frac{\pi}{2}\right)\right)$

$|A| = 2$

$\omega = \frac{1}{3}$

period $= \frac{2\pi}{\omega} = 6\pi$

phase shift $= \frac{\pi}{2}$



4. Find the exact value of each expression.

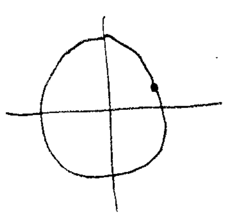
(a)

$$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$$

$\sec y = \frac{2}{\sqrt{3}}$ $\qquad 0 \le y \le \pi, \quad y \ne \frac{\pi}{2}$

$\cos y = \frac{\sqrt{3}}{2} \Rightarrow \boxed{y = \frac{\pi}{6}}$

(b)



$$\sin^{-1}\left(\sin \frac{11\pi}{5}\right) = y$$

$\sin y = \sin \frac{11\pi}{5}$, $\quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\boxed{y = \frac{\pi}{5}}$

(c)

$$\cot\left(\csc^{-1}\sqrt{10}\right)$$

$\csc^{-1}\sqrt{10} = \theta$

$\sqrt{10} = \csc\theta$ $\qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\sin\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} = \frac{y}{r}$ $\qquad y = \sqrt{10}$

$r = 10$

$\cot\theta = \frac{x}{y} = \frac{3\sqrt{10}}{\sqrt{10}} = \boxed{3}$ $\qquad x = \sqrt{90} = 3\sqrt{10}$

(d) $\cos 15°$ (Hint: what is $60° - 45°$?)

$= \cos(60° - 45°) = \cos 60° \cos 45° + \sin 60° \sin 45°$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

5. Establish each identity.

(a)

$$\sin\theta\tan\theta + \cos\theta = \sec\theta$$

$$\text{LHS} = \sin\theta\tan\theta + \cos\theta = \sin\theta\cdot\frac{\sin\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta = \text{RHS} \checkmark$$

(b)

$$\frac{\csc\theta + \cot\theta}{\sec\theta + \tan\theta} = \frac{\sec\theta - \tan\theta}{\csc\theta - \cot\theta}$$

$$\cancel{\frac{\csc\theta + \cot\theta}{\sec\theta + \tan\theta} = \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)}$$

scratch work

equivalent to: $(\csc\theta + \cot\theta)(\csc\theta - \cot\theta) = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$$(*) \quad \csc^2\theta - \cot^2\theta = \sec^2\theta - \tan^2\theta$$

use pyth. identities:

$$\text{LHS of } (*) = \csc^2\theta - \cot^2\theta = 1 + \cot^2\theta - \cot^2\theta$$

$$= 1$$

$$= 1 + \tan^2\theta - \tan^2\theta$$

$$= \sec^2\theta - \tan^2\theta = \text{RHS of } (*) \checkmark$$