

Name:

Class time:

M305G - Practice Final Exam - December 2008

Carefully read each question, and be sure to show your work.

1. (10 points) Consider the function $R(x) = x - \frac{1}{x}$.

(a) If we write $R(x) = \frac{p(x)}{q(x)}$, then what is the polynomial $p(x)$? And what is $q(x)$?

(b) What is the domain of $R(x)$? What is the y -intercept of the graph of $R(x)$?

(c) What are the x -intercepts of the graph of $R(x)$?

(d) What are the vertical asymptotes of the graph of $R(x)$?

(e) Does $R(x)$ have a horizontal or oblique asymptote? If so, what is it?

2. Let $f(x) = 2 + \log_3(x - 1)$. Note: f is a one-to-one function.

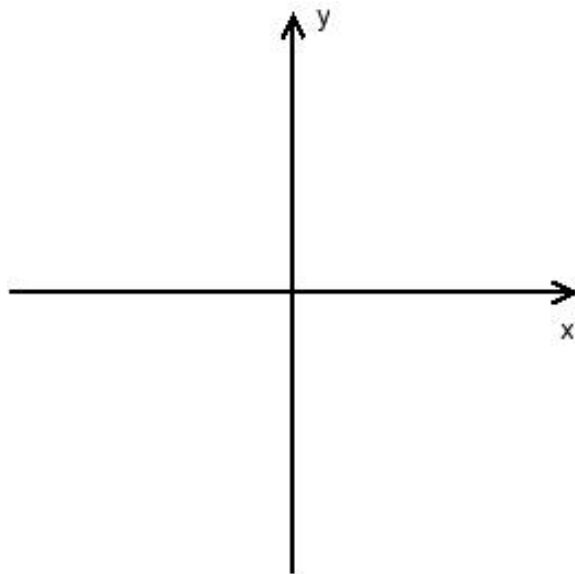
(a) What is $f(2)$? And what is $f(4)$?

(b) What is $f^{-1}(x)$?

(c) What is the domain of f ? And what is the range of f ?

(d) Using properties of logarithms, write $f(x)$ as a single logarithm. (Hint: What is $\log_3 9$?)

(e) Graph f and f^{-1} on the same xy -plane below. Clearly label both points you found to be on the graph of f in part 1 as well as any asymptotes.



3. (10 points) This problem involves piecewise-defined functions AND composite functions.

Let

$$f(x) = \begin{cases} 4x + 8 & x \geq -2 \\ x + 2 & x < -2 \end{cases}$$

and

$$g(x) = 2x + 1$$

(a) What is $(f \circ g)(1)$?

(b) What is $(g \circ f)(-3)$?

(c) Is f one-to-one? Why or why not?

(d) Consider the two cases $x \geq -2$ and $x < -2$ to solve

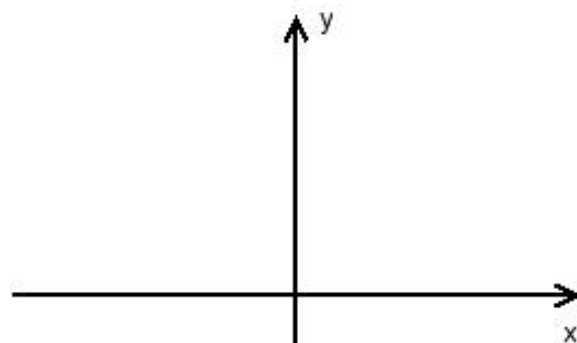
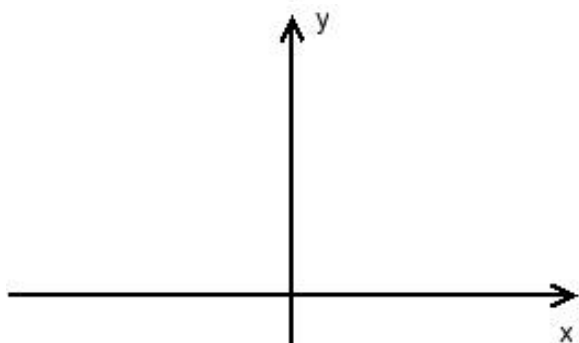
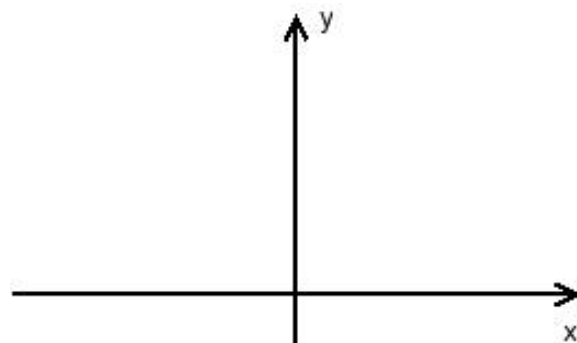
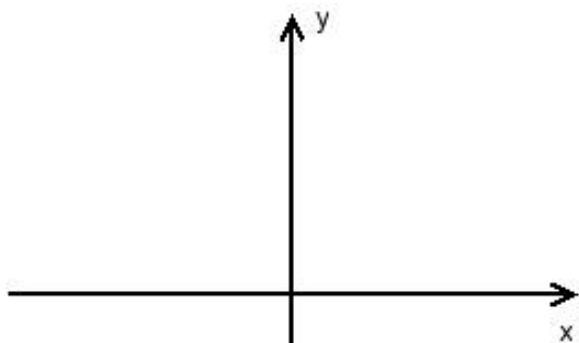
$$\frac{f(x)}{g(x)} \leq 0$$

4. (10 points) Let $g(x) = \frac{1}{2}|x + 1| - 2$.

(a) Write $g(x)$ as a piecewise-defined function:

$$g(x) = \left\{ \right.$$

(b) Starting with the graph of the basic function from your library of functions and showing all stages, graph the function $g(x)$ using the techniques of shifting, compressing, stretching, and/or reflecting. Label at least three points on each graph.



5. (10 points) Recall that a general quadratic function $f(x) = ax^2 + bx + c$ may be rewritten $f(x) = a(x - h)^2 + k$ for appropriate h and k . For this problem, let $g(x)$ be the quadratic function

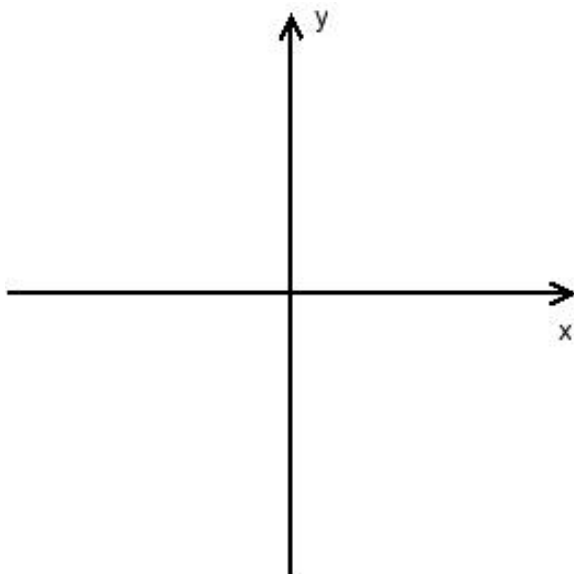
$$g(x) = 2x^2 - x - 3.$$

- (a) Find h and k for $g(x)$. Without drawing the graph of $g(x)$, determine its vertex and axis of symmetry.

- (b) Find the y -intercept of $g(x)$.

- (c) What are the x -intercepts of $g(x)$?

- (d) Graph $g(x)$ below. Label all points that you found in parts (a) through (c).



6. (10 points) In this problem, you will create and graph a polynomial function.

(a) Form a polynomial $p(x)$ having ALL OF THE FOLLOWING as real zeros:

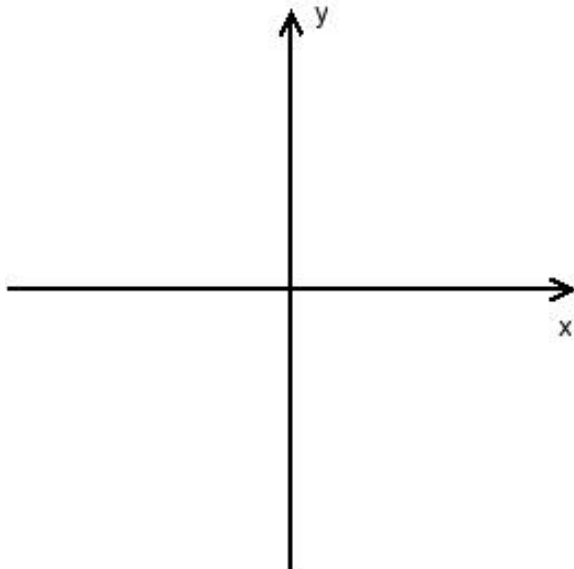
- $x = -2$ of multiplicity 2
- $x = 0$ of multiplicity 1
- $x = 3$ of multiplicity 2

(Note: there is more than one right answer for this problem.)

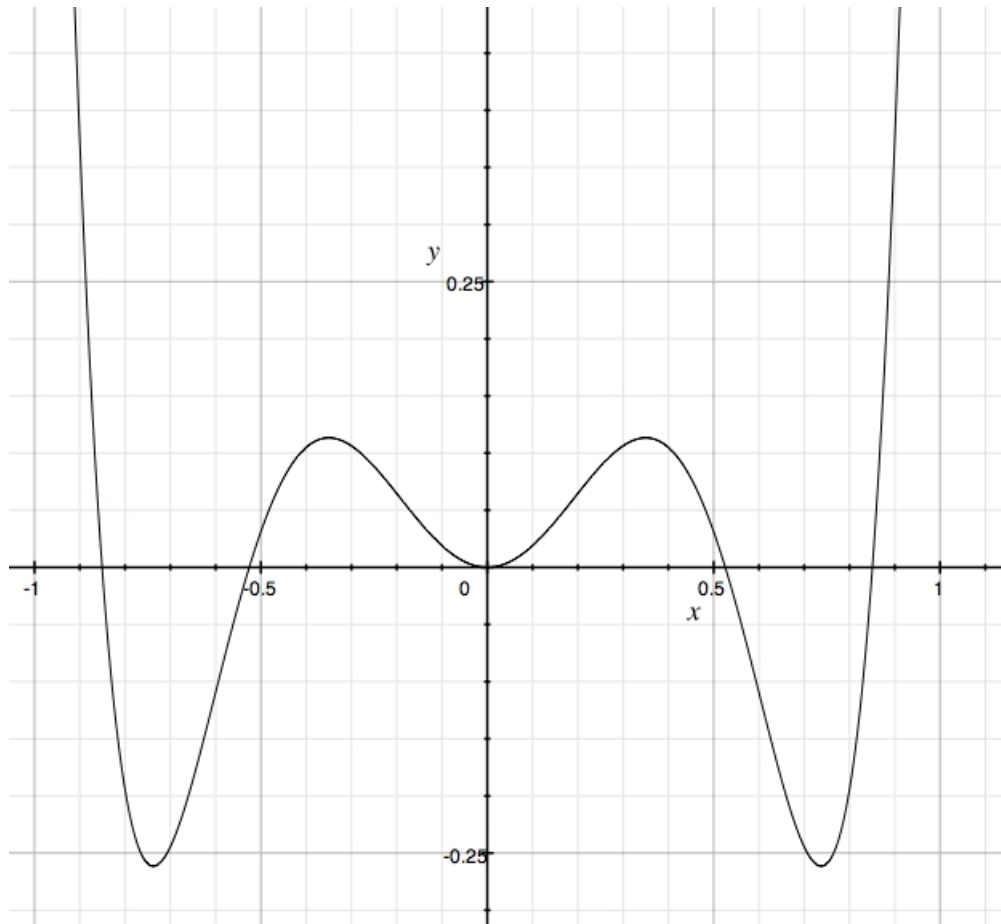
(b) What is the degree of your polynomial?

(c) What is the y -intercept of the graph of $p(x)$?

(d) Sketch the graph of your polynomial. Label all intercepts.



7. (10 points) Consider the graph of a degree 6 polynomial $f(x)$, which happens to be an even function:



For each of the following questions, estimate the answer using the information from the graph.

- What is $f(0.4)$?
- Does the graph of f have any symmetry? If so, what kind does it have?
- What are the x -coordinates of the local minima of this graph? What are the values of the function at each of them?
- On what intervals is this function increasing?
- On what interval(s) is $f(x) \leq 0$?

8. (10 points) Solve the following equations:

(a)

$$e^{4x} = e^{x^2} \cdot \frac{1}{e^4}$$

(b)

$$\sec\left(\sin^{-1}\frac{2\sqrt{5}}{5}\right)$$

(c)

$$\log_2(x - 1) + 2 \log_4(x - 4) = 2$$

(Hint: use the change of base formula to rewrite $\log_4(x - 4)$ in terms of \log_2 .)

(d)

$$3 \cdot 4^x - 4 \cdot 2^x = -1$$

9. (10 points) Establish each identity.

(a) $\frac{\cos(\alpha+\beta)}{\cos \alpha \cos \beta} = 1 - \tan \alpha \tan \beta$

(b) $\sec(2\theta) = \frac{\sec^2 \theta}{2 - \sec^2 \theta}$

10. Suppose you have a right triangle with two non-hypotenuse sides a and b of lengths $\sqrt{6} + \sqrt{2}$ and $\sqrt{6} - \sqrt{2}$ respectively, and let A and B denote the angles opposite sides a and b .

(a) What is the length of the hypotenuse, c ?

(b) What is the measure of angle A ?