This print-out should have 23 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering. The due time is Central time.

version 888

CalC8e58s

54:06, calculus3, multiple choice, > 1 min, wording-variable.

001

Determine the indefinite integral

$$I = \int 2x \ln(2+x)\, dx\,.$$

1. $I = (x^2-4)\ln(2+x) + 2x + \frac{1}{2}x^2 + C$

2. $I = \frac{1}{2}(x^2-4)\ln(2+x) - x - \frac{1}{4}x^2 + C$

3. $I = \frac{1}{2}(x^2-4)\ln(2+x) + x - \frac{1}{4}x^2 + C$

4. $I = 2(x^2-4)\ln(2+x) - 4x + x^2 + C$

5. $I = (x^2-4)\ln(2+x) + 2x - \frac{1}{2}x^2 + C$ **correct**

6. $I = 2(x^2-4)\ln(2+x) + 4x - x^2 + C$

Explanation:

Integrate first by parts. Then

$$I = x^2\ln(2+x) - \int \frac{x^2}{2+x}\, dx\,.$$

To evaluate the integral on the right we divide:

$$\frac{x^2}{2+x} = \frac{2x + x^2 - 2x}{2+x}$$

$$= x - \frac{2x}{2+x} = x - 2 + \frac{4}{2+x}\,.$$

In this case,

$$\int \frac{x^2}{2+x}\, dx = \int \left\{x - 2 + \frac{4}{2+x}\right\} dx$$

$$= \frac{1}{2}x^2 - 2x + 4\ln(2+x)\,.$$

Consequently,

$$\boxed{I = (x^2-4)\ln(2+x) + 2x - \frac{1}{2}x^2 + C}$$

with C an arbitrary constant.

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keywords: indefinite integral, integration by parts, natural log, polynomial long division

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CalC8a08d

54:02, calculus3, multiple choice, > 1 min, wording-variable.

002

Find the indefinite integral

$$\int 3x(\ln x)^2\, dx.$$

1. $\frac{3}{2}x^2\Big((\ln x)^2 + \ln x + \frac{1}{2}\Big) + C$

2. $6x^2\Big((\ln x)^2 - \ln x + \frac{1}{2}\Big) + C$

3. $\frac{3}{2}x^2\Big((\ln x)^2 - \ln x + \frac{1}{2}\Big) + C$ **correct**

4. $3x^2\Big((\ln x)^2 - \ln x + \frac{1}{2}\Big) + C$

5. $\frac{3}{2}x^2\Big((\ln x)^2 - \ln x - \frac{1}{2}\Big) + C$

Explanation:

After integration by parts,

$$\int x(\ln x)^2\, dx = \frac{1}{2}x^2(\ln x)^2 - \int x^2\frac{1}{x}\ln x\, dx$$

$$= \frac{1}{2}x^2(\ln x)^2 - \int x\ln x\, dx.$$

But after integration by parts once again,

$$\int x\ln x\, dx = \frac{1}{2}x^2\ln x - \frac{1}{2}\int x^2\frac{1}{x}\, dx$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{2}\int x\, dx$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C.$$

Thus

$$\int x(\ln x)^2\,dx = \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2 + C.$$

Consequently,

$$\int 3x(\ln x)^2\,dx = \boxed{\frac{3}{2}x^2\Big((\ln x)^2 - \ln x + \frac{1}{2}\Big) + C.}$$

keywords: integration by parts, log function

CalC8b22exam

54:03, calculus3, multiple choice, > 1 min, wording-variable.

003

Evaluate the definite integral

$$I = \int_0^{\pi/4} 4\tan^4 x\,dx\,.$$

1. $I = \pi - \dfrac{8}{3}$ **correct**

2. $I = 2\pi + \dfrac{4}{3}$

3. $I = 2\pi - \dfrac{8}{3}$

4. $I = \pi + \dfrac{8}{3}$

5. $I = 4\pi - \dfrac{4}{3}$

6. $I = 4\pi + \dfrac{4}{3}$

Explanation:

Since

$$\tan^2 x = \sec^2 x - 1\,,$$

it follows that

$$\tan^4 x = \tan^2 x(\sec^2 x - 1) = \tan^2 x\sec^2 - \tan^2 x\,.$$

Thus, using the same trig identity as before, we see that

$$\tan^4 x = (\tan^2 x - 1)\sec^2 x + 1\,,$$

in which case

$$I = 4\int_0^{\pi/4}\Big\{(\tan^2 x - 1)\sec^2 x + 1\Big\}\,dx\,.$$

The whole point of this use of trig identities is that

$$\frac{d}{dx}\tan x = \sec^2 x\,,$$

so set $u = \tan x$. Then

$$du = \sec^2 x\,dx\,,$$

while

$$x = 0 \implies u = 0\,,$$
$$x = \frac{\pi}{4} \implies u = 1\,.$$

In this case,

$$I = 4\int_0^1 (u^2 - 1)\,du + 4\Big[\,x\,\Big]_0^{\pi/4} = 4\Big[\frac{1}{3}u^3 - u\Big]_0^1 + \pi\,.$$

Consequently,

$$\boxed{I = \pi - \frac{8}{3}}\,.$$

keywords: trig identity, substitution, integral

CalC8b01exam

54:03, calculus3, multiple choice, < 1 min, wording-variable.

004

Evaluate the integral

$$I = \int_0^{\pi/2} \sin^3 x\,dx\,.$$

1. $I = 1$

2. $I = \frac{1}{6}$

3. $I = \frac{5}{6}$

4. $I = \frac{2}{3}$ **correct**

5. $I = \frac{1}{3}$

Explanation:
Since

$$\sin^2 x = 1 - \cos^2 x,$$

we see that

$$I = \int_0^{\pi/2} (1 - \cos^2 x) \sin x \, dx.$$

This suggests using the substitution $u = \cos x$. For then $du = -\sin x \, dx$, while

$$x = 0 \implies u = 1,$$

$$x = \frac{\pi}{2} \implies u = 0.$$

In this case,

$$I = -\int_1^0 (1 - u^2) \, du = \int_0^1 (1 - u^2) \, du.$$

Thus

$$\boxed{I = \left[u - \frac{1}{3}u^3 \right]_0^1 = \frac{2}{3}}.$$

keywords: definite integral, pythagorean identity, trig integral

CalC8c25exam

54:04, calculus3, multiple choice, > 1 min, wording-variable.

005

Evaluate the definite integral

$$I = \int_{\sqrt{2}}^{2} \frac{8}{x\sqrt{x^2 - 1}} \, dx.$$

1. $I = 1$

2. $I = \frac{4}{3}\pi$

3. $I = \frac{4}{3}$

4. $I = \pi$

5. $I = \frac{2}{3}$

6. $I = \frac{2}{3}\pi$ **correct**

Explanation:
Set $x = \sec u$. Then

$$dx = \sec u \tan u \, du, \quad x^2 - 1 = \tan^2 u,$$

while

$$x = \sqrt{2} \implies u = \frac{\pi}{4},$$

$$x = 2 \implies u = \frac{\pi}{3}.$$

In this case,

$$I = 8 \int_{\pi/4}^{\pi/3} \frac{\sec u \tan u}{\sec u \tan u} \, du = \int_{\pi/4}^{\pi/3} 8 \, du.$$

Consequently,

$$\boxed{I = 8\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{2}{3}\pi}.$$

keywords:

CalC8e16exam

54:06, calculus3, multiple choice, > 1 min, wording-variable.

006

Evaluate the definite integral

$$I = \int_0^{\frac{1}{\sqrt{2}}} \frac{2x^2}{\sqrt{1 - x^2}} \, dx.$$

1. $I = \frac{\pi}{2} - 1$

2. $I = \dfrac{\pi}{2} - \dfrac{1}{4}$

3. $I = \pi - \dfrac{1}{2}$

4. $I = \dfrac{\pi}{4} - \dfrac{1}{2}$ **correct**

5. $I = \dfrac{\pi}{4} - \dfrac{1}{8}$

6. $I = \dfrac{\pi}{8} - \dfrac{1}{4}$

Explanation:

Set $x = \sin u$. Then

$$dx = \cos u\, du, \qquad \sqrt{1-x^2} = \cos u\,,$$

while

$$x = 0 \implies u = 0,$$

$$x = \frac{1}{\sqrt{2}} \implies u = \frac{\pi}{4}.$$

In this case

$$I = \int_0^{\pi/4} \frac{2\sin^2 u \cos u}{\cos u}\, du$$

$$= 2\int_0^{\pi/4} \sin^2 u\, du$$

$$= \int_0^{\pi/4} \Big(1 - \cos 2u\Big)\, du\,.$$

Thus

$$I = \Big[u - \frac{1}{2}\sin 2u\Big]_0^{\pi/4}.$$

Consequently,

$$\boxed{I = \frac{1}{4}\pi - \frac{1}{2}}.$$

keywords: definite integral, trig. substitution, half-angle identity

CalC8c02c

54:04, calculus3, multiple choice, > 1 min, wording-variable.

007

Evaluate the integral

$$I = \int_0^{1/4} \frac{4}{\sqrt{1-4x^2}}\, dx\,.$$

1. $I = \dfrac{2}{3}\pi$

2. $I = \dfrac{1}{3}\pi$ **correct**

3. $I = \dfrac{1}{2}\pi$

4. $I = \dfrac{2}{3}$

5. $I = \dfrac{1}{3}$

6. $I = \dfrac{1}{2}$

Explanation:

Set $2x = 1\sin u$. Then

$$2\, dx = 1\cos u\, du$$

and

$$1 - 4x^2 = 1 - \sin^2 u = \cos^2 u\,,$$

while

$$x = 0 \implies u = 0,$$

$$x = \frac{1}{4} \implies u = \frac{\pi}{6}.$$

In this case,

$$I = \frac{1}{2}\int_0^{\pi/6} \frac{4\cos u}{\cos u}\, du = 2\int_0^{\pi/6} du\,.$$

Consequently,

$$\boxed{I = \frac{1}{3}\pi}.$$

keywords:

CalC8e04a

54:06, calculus3, multiple choice, $<$ 1 min, wording-variable.

008

Evaluate the definite integral

$$I = \int_0^1 (5+4x)^{-1/2}\,dx\,.$$

1. $I = \frac{1}{4}\left(3-\sqrt{5}\right)$

2. $I = \frac{1}{5}$

3. $I = \frac{1}{2}\left(3-2\right)$

4. $I = 0$

5. $I = \frac{2}{5}$

6. $I = \frac{1}{2}\left(3-\sqrt{5}\right)$ **correct**

Explanation:

Set $u = 5+4x$. Then $du = 4\,dx$, while

$$x = 0 \implies u = 5\,,$$

$$x = 1 \implies u = 9\,.$$

In this case

$$I = \frac{1}{4}\int_5^9 u^{-1/2}\,du = \left[\frac{1}{2}u^{1/2}\right]_5^9\,.$$

Consequently,

$$\boxed{I = \frac{1}{2}\left(3-\sqrt{5}\right)}\,.$$

keywords: substitution, fractional power

CalC8c32a

54:04, calculus3, multiple choice, $>$ 1 min, wording-variable.

009

Evaluate the definite integral

$$I = \int_{-3}^{-1} \frac{4}{\sqrt{7-6x-x^2}}\,dx\,.$$

1. $I = \frac{2}{3}\pi$ **correct**

2. $I = \sqrt{3}$

3. $I = \frac{2}{3}\sqrt{3}$

4. $I = \frac{4}{3}\sqrt{3}$

5. $I = \pi$

6. $I = \frac{4}{3}\pi$

Explanation:

By completing the square we see that

$$7-6x-x^2 = 16-(x+3)^2\,,$$

so

$$I = \int_{-3}^{-1} \frac{4}{\sqrt{16-(x+3)^2}}\,,dx\,.$$

Now set $x+3 = 4\sin u$. Then

$$dx = 4\cos u\,du\,,$$

while

$$x = -3 \implies u = 0,$$

$$x = -1 \implies u = \frac{\pi}{6}\,.$$

Thus

$$I = 4\int_0^{\pi/6} \frac{4\cos u}{4\cos u}\,du = \Big[4u\Big]_0^{\pi/6}\,.$$

Consequently,

$$\boxed{I = \frac{2}{3}\pi}\,.$$

keywords: definite integral, completion square, inverse sin integral

CalC8d10a

54:05, calculus3, multiple choice, > 1 min, wording-variable.

010

Evaluate the definite integral

$$I = \int_0^1 \frac{18}{2x^2 - 3x - 9}\, dx\,.$$

1. $I = 2\ln\frac{4}{5}$

2. $I = 4\ln\frac{4}{5}$

3. $I = 2\ln\frac{2}{5}$ **correct**

4. $I = 4\ln\frac{2}{5}$

5. $I = 2\ln\frac{3}{5}$

6. $I = 4\ln\frac{3}{5}$

Explanation:

By partial fractions,

$$\frac{18}{2x^2 - 3x - 9} = \frac{2}{x-3} - \frac{4}{2x+3}\,.$$

Thus

$$I = \int_0^1 \frac{2}{x-3}\, dx - \int_0^1 \frac{4}{2x+3}\, dx\,.$$

But

$$\int_0^1 \frac{2}{x-3}\, dx = \Big[2\ln|x-3|\Big]_0^1,$$

while

$$\int_0^1 \frac{4}{2x+3}\, dx = \Big[2\ln|2x+3|\Big]_0^1.$$

Consequently,

$$\boxed{I = \left[2\ln\left|\frac{x-3}{2x+3}\right|\right]_0^1 = 2\ln\frac{2}{5}}.$$

keywords: definite integral, rational function, partial fractions, natural log

CalC8g01c

51:06, calculus3, multiple choice, > 1 min, wording-variable.

011

If the points

$$(0,\, 3),\quad (\tfrac{1}{2},\, 5),\quad (1,\, 4),\quad (\tfrac{3}{2},\, 5),\quad (2,\, 3)$$

lie on the graph of a continuous function $y = f(x)$, use Simpson's Rule and all these points to estimate the definite integral

$$I = \int_0^2 f(x)\, dx.$$

1. $I \approx 9$ **correct**

2. $I \approx \frac{17}{2}$

3. $I \approx \frac{25}{3}$

4. $I \approx \frac{53}{6}$

5. $I \approx \frac{26}{3}$

Explanation:

Simpson's Rule estimates the definite integral I as

$$\frac{h}{3}\Big(f(0) + 4f(\tfrac{1}{2}) + 2f(1) + 4f(\tfrac{3}{2}) + f(2)\Big).$$

With $h = \frac{1}{2}$ and the given values of f, therefore, the area is estimated by

$$\boxed{I \approx 9.}$$

keywords: Simpson's Rule, integral, graph

CalC8g07a

51:06, calculus3, multiple choice, > 1 min, wording-variable.

012

After partitioning the interval [0, 4] into 4 equal subintervals, use the trapezoidal rule to estimate the integral

$$I = \int_0^4 \frac{7}{\sqrt{1+x^2}}\,dx\,.$$

1. $I \approx 15.2427$
2. $I \approx 14.4427$
3. $I \approx 14.8427$
4. $I \approx 14.6427$ **correct**
5. $I \approx 15.0427$

Explanation:

When the interval [0, 4] is partitioned into 4 equal subintervals the trapezoidal rule estimates the integral

$$I = \int_0^4 f(x)\,dx$$

as

$$I \approx \frac{1}{2}\Big(f(0)+2f(1)+2f(2)+2f(3)+f(4)\Big).$$

When

$$f(x) = \frac{7}{\sqrt{1+x^2}},$$

therefore,

$$I \approx \frac{7}{2}\Big(1+\frac{2}{\sqrt{1+1}}+\frac{2}{\sqrt{1+4}} + \frac{2}{\sqrt{1+9}}+\frac{1}{\sqrt{1+16}}\Big)\,.$$

Consequently,

$$\boxed{I \approx 14.6427}\,.$$

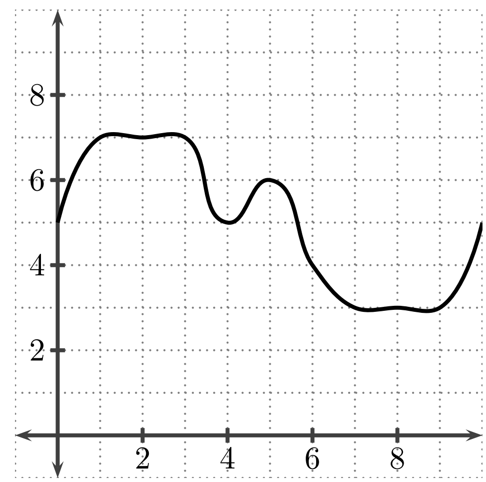
keywords: partition, trapezoidal rule, integral, estimate

CalC8g01d

51:06, calculus3, multiple choice, < 1 min, wording-variable.

013

If f is the function whose graph on [0, 10] is given by



use the Trapezoidal Rule with $n = 5$ to estimate the definite integral

$$I = \int_3^8 f(x)\,dx\,.$$

1. $I \approx 23$ **correct**
2. $I \approx \frac{47}{2}$
3. $I \approx \frac{43}{2}$
4. $I \approx \frac{45}{2}$
5. $I \approx 22$

Explanation:

The Trapezoidal Rule estimates the definite integral

$$I = \int_3^8 f(x)\,dx$$

by

$$I \approx \frac{1}{2}\Big[f(3)+2\{f(4)+ \cdots + f(7)\}+f(8)\Big]$$

when $n = 5$. For the given f, therefore,

$$\boxed{I \approx \frac{1}{2}\Big[7 + 2\{5 + 6 + 4 + 3\} + 3\Big] = 23},$$

reading off the values of f from the graph.

keywords: trapezoidal rule, integral, graph

CalC8h21b

54:08, calculus3, multiple choice, > 1 min, normal.

014

Determine if the improper integral

$$I = \int_e^\infty \frac{1}{x(\ln 3x)^2}\,dx$$

converges, and if it does, compute its value.

1. $I = \ln 3e$

2. $I = \dfrac{3}{\ln 3e}$

3. $I = \dfrac{1}{3e}$

4. $I = 3$

5. $I = \dfrac{1}{\ln 3e}$ **correct**

6. I does not converge

Explanation:

The integral is improper because of the infinite interval of integration, so we write

$$I = \lim_{t\to\infty} I_t, \quad I_t = \int_e^t \frac{1}{x(\ln 3x)^2}\,dx,$$

whenever the limit exists. To evaluate I_t, first set $u = \ln 3x$. Then

$$\int \frac{1}{x(\ln 3x)^2}\,dx = \int \frac{1}{u^2}\,du = -\frac{1}{u} + C,$$

and so

$$I_t = \Big[-\frac{1}{\ln 3x}\Big]_e^t = \Big(\frac{1}{\ln 3e} - \frac{1}{\ln 3t}\Big).$$

On the other hand,

$$\lim_{t\to\infty} \frac{1}{\ln 3t} = 0.$$

Consequently, $\lim_{t\to\infty} I_t$ exists, and

$$\boxed{I = \lim_{t\to\infty}\Big(\frac{1}{\ln 3e} - \frac{1}{\ln 3t}\Big) = \frac{1}{\ln 3e}}.$$

keywords: improper integral, limit, log function, infinite interval integration,

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CalC8h07s

54:08, calculus3, multiple choice, > 1 min, wording-variable.

015

Determine if the improper integral

$$I = \int_{-\infty}^5 \frac{1}{\sqrt{6-z}}\,dz$$

is convergent or divergent, and if convergent, find its value.

1. $I = \dfrac{1}{2}$

2. $I = \dfrac{3}{2}$

3. $I = 1$

4. I is divergent **correct**

5. $I = 2$

Explanation:

The integral is improper because of the infinite interval of integration. It will converge if

$$\lim_{t\to\infty} \int_{-t}^5 \frac{1}{\sqrt{6-z}}\,dz$$

exists. Now

$$\int_{-t}^5 \frac{1}{\sqrt{6-z}}\,dz = \Big[-2\sqrt{6-z}\Big]_{-t}^5$$
$$= -2 + 2\sqrt{6+t}.$$

But we know that

$$\lim_{t \to \infty} \sqrt{6+t} = \infty .$$

Consequently,

$$\boxed{I \text{ is divergent}} .$$

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keywords: improper integral, substitution, infinite interval integration, convergent, divergent

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CalC8h40s

54:08, calculus3, multiple choice, > 1 min, wording-variable.

016

Determine if the improper integral

$$I = \int_0^1 \frac{4\ln 7x}{\sqrt{x}}\, dx$$

is convergent or divergent, and if convergent, find its value.

1. $I = 8(\ln 7 + 2)$
2. $I = 4(\ln 7 + 1)$
3. I is divergent
4. $I = 7(\ln 7 + 1)$
5. $I = 7(\ln 7 - 1)$
6. $I = 4(\ln 7 - 2)$
7. $I = 8(\ln 7 - 2)$ **correct**

Explanation:

The integral is improper because the integrand has a vertical asymptote at $x = 0$, and so is not bounded on [0, 1]. Thus

$$I = \lim_{t \to 0+} \int_t^1 \frac{4\ln 7x}{\sqrt{x}}\, dx .$$

To evaluate the integral

$$I_t = \int_t^1 \frac{4\ln 7x}{\sqrt{x}}\, dx$$

we use Integration by Parts. For then

$$\begin{aligned} I_t &= 8\Big\{\Big[\sqrt{x}\ln 7x\Big]_t^1 - \int_t^1 \frac{\sqrt{x}}{x}\, dx\Big\} \\ &= 8\Big\{\Big[\sqrt{x}\ln 7x\Big]_t^1 - \int_t^1 \frac{1}{\sqrt{x}}\, dx\Big\} \\ &= 8\Big[\sqrt{x}\ln 7x - 2\sqrt{x}\Big]_t^1 . \end{aligned}$$

Thus

$$I_t = 8(\ln 7 - 2) - 8(\sqrt{t}\ln 7t - 2\sqrt{t}) .$$

But by L'Hospital's Rule,

$$\lim_{t \to 0+} \sqrt{t}\ln 7t = 0 .$$

Consequently,

$$\boxed{I = 8(\ln 7 - 2)} .$$

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keywords: improper integral, logarithmic function, convergent, divergent, L'Hospital's Rule, substitution

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CalC8h57exam

54:08, calculus3, multiple choice, < 1 min, wording-variable.

017

Determine if the integral

$$I = \int_0^1 \frac{4 + 3x^{-1/3}}{x^{1/2}}\, dx$$

converges, and if it does find its value.

1. $I = 27$
2. $I = 26$ **correct**
3. $I = 25$

4. I does not converge

5. $I = 29$

6. $I = 28$

Explanation:

In the given integral,

$$\frac{4+3x^{-1/3}}{x^{1/2}} = \frac{4}{x^{1/2}} + \frac{3}{x^{5/6}}$$

for $x > 0$, so it is enough to check if the integral

$$I_p = \int_0^1 \frac{1}{x^p}\,dx$$

converges for $p < 1$.

Now I_p will converge when the limit

$$\lim_{t\to 0+} \int_t^1 \frac{1}{x^p}\,dx$$

exists. But

$$\int_t^1 \frac{1}{x^p}\,dx = \Big[\frac{x^{-p+1}}{-p+1}\Big]_t^1$$

$$= \frac{1}{1-p}\Big(1 - \frac{1}{t^{p-1}}\Big).$$

If $p < 1$, then $p - 1 < 0$, so

$$\lim_{t\to 0+} \frac{1}{t^{p-1}} = \lim_{t\to 0+} t^{1-p} = 0.$$

Thus I_p converges for all $p < 1$ and

$$I_p = \lim_{t\to 0+} \frac{1}{1-p}\big(1 - t^{1-p}\big)\Big) = \frac{1}{1-p}.$$

Since both exponents satisfy the condition $p < 1$, I converges and

$$\boxed{I = \frac{4}{1/2} + \frac{3}{1/6} = 26}.$$

keywords: improper integral, power function, exponents

CalC15c11a

59:03, calculus3, multiple choice, < 1 min, wording-variable.

018

Determine f_x when

$$f(x,\,y) = (xy-3)e^{-xy}.$$

1. $f_x = -y(2+xy)e^{-xy}$

2. $f_x = -x(2+3xy)e^{-xy}$

3. $f_x = x(4-3xy)e^{-xy}$

4. $f_x = y(4-xy)e^{-xy}$ **correct**

5. $f_x = -x(2+xy)e^{-xy}$

6. $f_x = x(3xy-4)e^{-xy}$

7. $f_x = -y(2-xy)e^{-xy}$

8. $f_x = y(xy-4)e^{-xy}$

Explanation:

From the Product Rule we see that

$$f_x = ye^{-xy} - y(xy-3)e^{-xy}.$$

Consequently,

$$\boxed{f_x = y(4-xy)e^{-xy}}.$$

keywords: partial derivative, first order partial derivative, exp function,

CalC15c10a

59:03, calculus3, multiple choice, > 1 min, normal.

019

Find the slope in the x-direction at the point $P(0, 2, f(0, 2))$ on the graph of f when

$$f(x, y) = 3(2x+y)e^{-xy}.$$

1. slope $= -12$

2. slope $= -8$

3. slope $= -6$ **correct**

4. slope $= -10$

5. slope $= -4$

Explanation:

The graph of f is a surface in 3-space and the slope in the x-direction at the point $P(0, 2, f(0, 2))$ on that surface is the value of the partial derivative f_x at (0, 2). Now

$$f_x = 6e^{-xy} - 3(2xy + y^2)e^{-xy}.$$

Consequently, at $P(0, 2, f(0, 2))$

$$\boxed{\text{slope} = -6}.$$

keywords: partial differentiation, slope, exp function

CalC15c47a

59:03, calculus3, multiple choice, > 1 min, wording-variable.

020

Find the value of $f_{xx} + f_{yy}$ at $(1, -1)$ when

$$f(x, y) = \frac{2}{xy} + 3x^2 + 4y^2.$$

1. $(f_{xx} + f_{yy})\Big|_{(1,-1)} = 5$

2. $(f_{xx} + f_{yy})\Big|_{(1,-1)} = 7$

3. $(f_{xx} + f_{yy})\Big|_{(1,-1)} = 6$ **correct**

4. $(f_{xx} + f_{yy})\Big|_{(1,-1)} = 23$

5. $(f_{xx} + f_{yy})\Big|_{(1,-1)} = 22$

Explanation:

Differentiating f twice with respect to x we obtain

$$\frac{\partial f}{\partial x} = -\frac{2}{x^2 y} + 6x, \quad \frac{\partial^2 f}{\partial x^2} = \frac{4}{x^3 y} + 6.$$

Repeating for y we next obtain

$$\frac{\partial f}{\partial y} = -\frac{2}{xy^2} + 8y, \quad \frac{\partial^2 f}{\partial x^2} = \frac{4}{xy^3} + 8.$$

Thus at $(1, -1)$,

$$\boxed{(f_{xx} + f_{yy})\Big|_{(1,-1)} = 6.}$$

keywords:

CalC16b19a

60:01, calculus3, multiple choice, > 1 min, wording-variable.

021

Evaluate the integral, I, of the function

$$f(x, y) = 2xe^{2xy}$$

over the rectangle

$$A = \{(x, y) : 0 \le x \le 3, \ 0 \le y \le 2\}.$$

1. $I = \frac{1}{4}\Big(e^{12} - 11\Big)$

2. $I = \frac{1}{8}\Big(e^{12} - 13\Big)$

3. $I = \frac{1}{8}\Big(e^{12} - 11\Big)$

4. $I = \frac{1}{8}\Big(e^{12} - 12\Big)$

5. $I = \frac{1}{4}\Big(e^{12} - 13\Big)$ **correct**

6. $I = \frac{1}{4}\Big(e^{12} - 12\Big)$

Explanation:

The integral is given by

$$I = \int\int_A 2xe^{2xy}\, dxdy.$$

Since the integral with respect to y can be evaluated easily using substitution (or directly making the substitution in one's head), while the integral with respect to x requires integration by parts, this suggests that we should represent the double integral as the repeated integral

$$I = \int_0^3 \left(\int_0^2 2xe^{2xy}\,dy\right)dx.$$

Now after integration the inner integral becomes

$$\left[e^{2xy}\right]_0^2 = \left(e^{4x} - 1\right).$$

Thus

$$I = \int_0^3 (e^{4x} - 1)\,dx = \left[\frac{e^{4x}}{4} - x\right]_0^3,$$

and so

$$\boxed{I = \frac{1}{4}\left(e^{12} - 13\right)}.$$

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keywords:

CalC16c30a

60:02, calculus3, multiple choice, < 1 min, normal.

022

Evaluate the double integral

$$I = \int\int_D \frac{2}{(1+x+y)^{3/2}}\,dA$$

when D is the region in the first quadrant bounded by

$$y = 3x+3, \qquad x = 3$$

as well as the x and y-axes.

1. $I = 7$

2. $I = 4$ **correct**

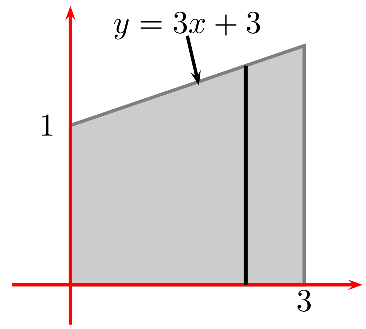
3. $I = 6$

4. $I = 5$

5. $I = 8$

Explanation:

The region of integration D is the region shaded in



the vertical line interior to the region showing that we should integrate first with respect to y. Then I becomes the repeated integral

$$I = \int_0^3 \left(\int_0^{3x+3} \frac{2}{(1+x+y)^{3/2}}\,dy\right)dx.$$

Now the inner integral is

$$\left[-\frac{4}{(1+x+y)^{1/2}}\right]_0^{3x+3} = \frac{2}{(1+x)^{1/2}},$$

and so

$$I = \int_0^3 \frac{2}{(1+x)^{1/2}}\,dx = \left[4(1+x)^{1/2}\right]_0^3.$$

Consequently,

$$\boxed{I = 4}.$$

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keywords: double integral, non-rectangular region integration, radical function,

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CalC16c40s

60:02, calculus3, multiple choice, < 1 min, wording-variable.

023

Reverse the order of integration in the integral

$$I = \int_{-2}^1 \left(\int_{-\sqrt{y+2}}^0 f(x, y)\,dx\right)dy,$$

but make no attempt to evaluate either integral.

1. $I = \int_{-\sqrt{3}}^{0}\left(\int_{x^2-2}^{1} f(x,y)\,dy\right)dx$ **correct**

2. $I = \int_{0}^{\sqrt{3}}\left(\int_{x^2-2}^{1} f(x,y)\,dy\right)dx$

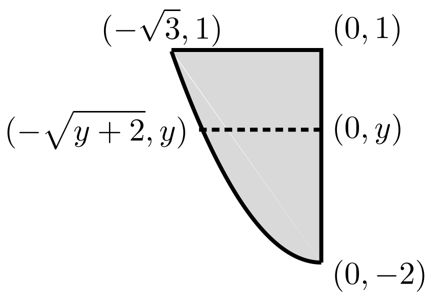
3. $I = \int_{0}^{\sqrt{3}}\left(\int_{1}^{x^2-2} f(x,y)\,dy\right)dx$

4. $I = \int_{-\sqrt{3}}^{0}\left(\int_{1}^{x^2+2} f(x,y)\,dy\right)dx$

5. $I = \int_{-\sqrt{3}}^{0}\left(\int_{x^2+2}^{1} f(x,y)\,dy\right)dx$

Explanation:

The region of integration is similar to the shaded region in the figure



(not drawn to scale) enclosed by the graphs of

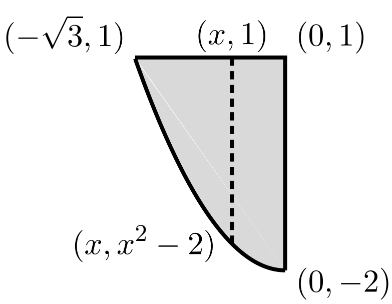
$$y = x^2 - 2, \quad y = 1, \quad x = 0,$$

and having corner points at

$$(-\sqrt{3}, 1), \quad (0, 1), \quad (0, -2).$$

Integration is taken first with respect to x for fixed y along the dashed horizontal line.

To change the order of integration, now fix x and let y vary along the dashed vertical line in



from (x, x^2-2) to $(x, 1)$. As the graph makes clear, x then varies from $-\sqrt{3}$ to 0. Hence, after changing the order of integration,

$$\boxed{I = \int_{-\sqrt{3}}^{0}\left(\int_{x^2-2}^{1} f(x,y)\,dy\right)dx}.$$

keywords: