This print-out should have 40 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering. The due time is Central time.

version 728

CalC12c04a

55:03, calculus3, multiple choice, > 1 min, wording-variable.

001

Determine which of the following series are convergent:

A. $$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

B. $$\sum_{n=1}^{\infty} \frac{3}{n^2+1}$$

C. $$\sum_{n=1}^{\infty} \frac{2}{n^{3/2}}$$

1. C only

2. A and C only

3. all of them **correct**

4. none of them

5. B and C only

6. A only

7. B only

8. A and B only

Explanation:

A. Series is $\sum_{n=1}^{\infty} \frac{1}{n^2}$. Use $f(x) = \frac{1}{x^2}$. Then

$$\int_1^{\infty} f(x)\,dx$$

convergent.

B. Use $f(x) = \frac{3}{x^2+1}$. Then

$$\int_1^{\infty} f(x)\,dx$$

convergent ($\tan^{-1}$ integral).

C. Use $f(x) = \frac{2}{x^{3/2}}$. Then

$$\int_1^{\infty} f(x)\,dx$$

convergent.

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keywords:

CalC12c15s

55:03, calculus3, multiple choice, < 1 min, wording-variable.

002

Determine whether the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2+4}$$

converges or diverges.

1. series diverges

2. series converges **correct**

Explanation:

We apply the integral test with

$$f(x) = \frac{1}{x^2+4}.$$

Now f is continuous, positive and decreasing on $[0, \infty)$. Thus the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2+4}$$

converges if and only if the improper integral

$$\int_0^\infty \frac{1}{x^2+4}\,dx$$

converges. But

$$\int_0^\infty \frac{1}{x^2+4}\,dx = \lim_{t\to\infty}\int_0^t \frac{1}{x^2+4}\,dx\,.$$

To evaluate this last integral, set $x = 2\tan u$. Then

$$dx = 2\sec^2 u\,du$$

while

$$x = 0 \implies u = 0,$$

$$x = t \implies u = \tan^{-1}\frac{t}{2}\,.$$

In this case

$$\int_0^t \frac{1}{x^2+4}\,dx = \frac{1}{2}\int_0^{\tan^{-1}(t/2)} \frac{\sec^2 u}{\sec^2 u}\,du$$

$$= \frac{1}{2}\Big[u\Big]_0^{\tan^{-1}(t/2)} = \frac{1}{2}\tan^{-1}\frac{t}{2}\,.$$

But by properties of $\tan^{-1} t$ we know that

$$\lim_{t\to\infty}\tan^{-1}\frac{t}{2} = \frac{\pi}{2}\,.$$

Consequently, by the Integral Test, the series

$$\boxed{\sum_{n=1}^\infty \frac{1}{n^2+4}\ \text{converges}}\,.$$

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keywords:

CalC12c20a

55:03, calculus3, multiple choice, < 1 min, wording-variable.

003

Determine whether the series

$$\sum_{m=1}^\infty \frac{3\ln(4m)}{m^2}$$

is convergent or divergent.

1. series converges **correct**
2. series diverges

Explanation:

The function

$$f(x) = \frac{3\ln(4x)}{x^2}$$

is continous and positive on $[\frac{1}{2}, \infty)$; in addition, since

$$f'(x) = 3\left(\frac{1-2\ln 4x}{x^3}\right) < 0$$

on $[\frac{1}{2}, \infty)$, f is also decreasing on this interval. This suggests applying the Integral Test, for then the series

$$\sum_{m=1}^\infty \frac{3\ln(4m)}{m^2}$$

is convergent if and only if the improper integral

$$\int_1^\infty f(x)\,dx = \int_1^\infty \frac{3\ln(4x)}{x^2}\,dx$$

converges.

Now, after Integration by Parts, we see that

$$\int_1^t f(x)\,dx = 3\Big[-\frac{\ln(4x)}{x} - \frac{1}{x}\Big]_1^t$$

$$= 3\Big\{-\frac{\ln(4t)}{t} - \frac{1}{t} + \ln 4 + 1\Big\}\,.$$

Consequently,

$$\int_1^\infty f(x)\,dx = \lim_{t\to\infty}\int_1^\infty f(x)\,dx$$

$$= \lim_{t\to\infty} 3\Big\{-\frac{\ln(4t)}{t} - \frac{1}{t} + \ln 4 + 1\Big\}$$

$$= 3(1+\ln 4)\,.$$

The Integral Test thus ensures that the given

$$\boxed{\text{series converges}}\,.$$

keywords:

CalC12d15s

55:04, calculus3, multiple choice, > 1 min, wording-variable.

004

Determine whether the series

$$\sum_{k=1}^{\infty} \frac{6+\cos k}{2^k}$$

converges or diverges.

1. series is convergent **correct**

2. series is divergent

Explanation:

We use the Comparison Test with

$$a_k = \frac{6+\cos k}{2^k}, \qquad b_k = \frac{7}{2^k}.$$

For then

$$0 < a_k \le b_k,$$

since $-1 \le \cos k \le 1$. Thus the series

$$\sum_{k=1}^{\infty} \frac{6+\cos k}{2^k}$$

converges if the series

$$\sum_{k=1}^{\infty} \frac{7}{2^k}$$

converges. But this last series is a geometric series with

$$|r| = \frac{1}{2} < 1,$$

hence convergent. Consequently, the given

$$\boxed{\text{series is convergent}}.$$

keywords: comparison tests, geometric series

CalC12d19b

55:04, calculus3, multiple choice, > 1 min, wording-variable.

005

Which of the following infinite series converges?

1. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5n}{7n+4}\right)^n$ **correct**

2. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{5n-4}$

3. $\displaystyle\sum_{k=2}^{\infty} \frac{4}{7k\ln k+5k}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{5n^n}{(n+5)^n}$

5. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5}{4}\right)^n$

Explanation:

We test the convergence of each of the five series.

(i) For the series

$$\sum_{k=2}^{\infty} \frac{4}{7k\ln k+5k}$$

the limit comparison test can be used, comparing it with

$$\sum_{k=2}^{\infty} \frac{1}{k\ln k}.$$

Now, after division,

$$k\ln k\left(\frac{4}{7k\ln k+5k}\right) = \frac{4}{7+\dfrac{5}{\ln k}}.$$

Since

$$\frac{4}{7+\dfrac{5}{\ln k}} \longrightarrow \frac{4}{7} > 0$$

as $k \to \infty$, the limit comparison test applies. But by the Integral test, the series

$$\sum_{k=2}^{\infty} \frac{1}{k\ln k}$$

does not converge. Thus

$$\sum_{k=2}^{\infty} \frac{4}{7k\ln k + 5k}$$

does not converge.

(ii) If a series $\sum_n a_n$ converges, then

$$a_n \longrightarrow 0$$

as $n \to \infty$. But for the series

$$\sum_{n=1}^{\infty} \frac{5n^n}{(n+5)^n}$$

we see that

$$a_n = \frac{5n^n}{(n+5)^n} = 5\left(\frac{n}{n+5}\right)^n$$

$$= 5\left(\frac{n+5}{n}\right)^{-n} = 5\left\{\left(1+\frac{5}{n}\right)^n\right\}^{-1}.$$

Now

$$\left(1+\frac{5}{n}\right)^n \longrightarrow e^5$$

as $n \to \infty$. Thus

$$\lim_{n\to\infty} a_n = 5e^{-5} \neq 0.$$

Consequently,

$$\sum_{n=1}^{\infty} \frac{5n^n}{(n+5)^n}$$

does not converge.

(iii) For the series

$$\sum_{n=1}^{\infty} \frac{7}{5n-4}$$

the comparison test can be used since

$$\frac{7}{5n-4} \geq \frac{7}{5}\left(\frac{1}{n}\right)$$

while

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

does not converge because of the Inegral test. Thus

$$\sum_{n=1}^{\infty} \frac{7}{5n-4}$$

does not converge.

(iv) Use of the Comparison test is suggested in dealing with

$$\sum_{n=1}^{\infty} \left(\frac{5n}{7n+4}\right)^n$$

for after division

$$\frac{5n}{7n+4} = \frac{5}{7+\dfrac{4}{n}}.$$

Now the inequality

$$0 < \frac{5}{7+\dfrac{4}{n}} \leq \frac{5}{7}$$

holds for all $n \geq 1$, so the inequality

$$\left(\frac{5n}{7n+4}\right)^n \leq \left(\frac{5}{7}\right)^n$$

holds for all $n \geq 1$. But

$$\sum_{n=1}^{\infty} \left(\frac{5}{7}\right)^n$$

is a geometric series whose common ratio is $r = \frac{5}{7}$. Since $0 < r < 1$, this geometric series converges. Hence by the Comparison test the series

$$\sum_{n=1}^{\infty} \left(\frac{5n}{7n+4}\right)^n$$

converges.

(v) The series

$$\sum_{n=1}^{\infty} \left(\frac{5}{4}\right)^n$$

is a geometric series with $r = \frac{5}{4} > 1$. But then this geometric series does not converge.

Consequently, of the five given infinite series, only

$$\boxed{\sum_{n=1}^{\infty} \left(\frac{5n}{7n+4}\right)^n}$$

converges.

keywords:

CalC12d34b
55:04, calculus3, multiple choice, > 1 min, wording-variable.

006

Which of the following series converge(s)?

$$(A) \quad \sum_{k=2}^{\infty} \frac{4k+3}{(k\ln k)^2+6}$$

$$(B) \quad \sum_{n=1}^{\infty} \frac{\sqrt{n}-7}{\sqrt{n}+3}$$

$$(C) \quad \sum_{n=1}^{\infty} \left(\frac{7n+3}{6n-4}\right)^n$$

1. C only
2. none of A, B, or C
3. A, B, and C
4. B and C
5. A and C
6. B only
7. A and B
8. A only **correct**

Explanation:

(A) After division,

$$\frac{4k+3}{(k\ln k)^2+6} = \frac{4+\dfrac{3}{k}}{k(\ln k)^2+\dfrac{6}{k}}.$$

But

$$k(\ln k)^2 \left(\frac{4+\dfrac{3}{k}}{k(\ln k)^2+\dfrac{6}{k}}\right) \longrightarrow 4 > 0,$$

so

$$k(\ln k)^2 \left(\frac{4k+3}{(k\ln k)^2+6}\right) \longrightarrow 4 > 0.$$

On the other hand, by the Integral test the infinite series

$$\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$$

converges, hence by the limit comparison test, the given series in (A) converges also.

(B) If an infinite series $\sum_n a_n$ converges, then

$$\lim_{n\to\infty} a_n = 0.$$

But for the given series in (B),

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sqrt{n}-7}{\sqrt{n}+3} = 1.$$

Consequently, the series in (B) does not converge.

(C) After division,

$$\frac{7n+3}{6n-4} = \frac{7+\dfrac{3}{n}}{6-\dfrac{4}{n}},$$

so the inequality

$$\left(\frac{7n+3}{6n-4}\right)^n \geq \left(\frac{7}{6}\right)^n$$

holds for all n. But the series

$$\sum_{n=1}^{\infty} \left(\frac{7}{6}\right)^n$$

is a geometric series whose common ratio $r = \frac{7}{6}$. Now $r > 1$, so this geometric series does not converge. Hence by the comparison test the series in (C) does not converge.

Consequently, of the given infinite series,

$$\boxed{\text{only } A}$$

converges.

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keywords: infinite series, comparison test, integral test, convergence series, divergence series

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CalC12e05s

55:05, calculus3, multiple choice, > 1 min, normal.

007

Which one of the following series is convergent?

1. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{5+\sqrt{n}}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{1+\sqrt{n}}$ **correct**

3. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{1+\sqrt{n}}{5+\sqrt{n}}$

4. $\displaystyle\sum_{n=1}^{\infty} (-1)^{2n}\frac{5}{3+\sqrt{n}}$

5. $\displaystyle\sum_{n=1}^{\infty} (-1)^{3}\frac{5}{3+\sqrt{n}}$

Explanation:

Since

$$\sum_{n=1}^{\infty} (-1)^3\frac{5}{3+\sqrt{n}} = -\sum_{n=1}^{\infty} \frac{5}{3+\sqrt{n}},$$

use of the Limit Comparison and p-series Tests with $p = \frac{1}{2}$ shows that this series is divergent. Similarly, since

$$\sum_{n=1}^{\infty} (-1)^{2n}\frac{5}{3+\sqrt{n}} = \sum_{n=1}^{\infty} \frac{5}{3+\sqrt{n}},$$

the same argument shows that this series as well as

$$\sum_{n=1}^{\infty} \frac{3}{5+\sqrt{n}}$$

is divergent.

On the other hand, by the Divergence Test, the series

$$\sum_{n=1}^{\infty} (-1)^{n-1}\frac{1+\sqrt{n}}{5+\sqrt{n}}$$

is divergent because

$$\lim_{n\to\infty} (-1)^{n-1}\frac{1+\sqrt{n}}{5+\sqrt{n}} \neq 0\,.$$

This leaves only the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{1+\sqrt{n}}\,.$$

To see that this series is convergent, set

$$b_n = \frac{1}{1+\sqrt{n}}\,.$$

Then

$$\text{(i)}\ \ b_{n+1} \leq b_n, \quad \text{(ii)}\ \ \lim_{n\to\infty} b_n = 0\,.$$

Consequently, by the Alternating Series Test, the series

$$\boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{1+\sqrt{n}}}$$

is convergent.

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keywords:

CalC12f04a

55:06, calculus3, multiple choice, < 1 min, normal.

008

Determine whether the series

$$\frac{4}{8} - \frac{4}{9} + \frac{4}{10} - \frac{4}{11} + \frac{4}{12} - \dots$$

is conditionally convergent, absolutely convergent, or divergent.

1. absolutely convergent

2. series is divergent

3. conditionally convergent **correct**

Explanation:

In summation notation,

$$\frac{4}{8} - \frac{4}{9} + \frac{4}{10} - \frac{4}{11} + \ldots = \sum_{n=4}^{\infty} (-1)^n f(n),$$

with

$$f(x) = \frac{4}{x+4}.$$

Now f is positive and decreasing on $[1, \infty)$. By the Integral test the given series will thus be absolutely convergent if the integral

$$\int_4^{\infty} f(x)\, dx$$

converges. However,

$$\int_4^t \frac{4}{x+4}\, dx = 4\Big[\ln(x+4)\Big]_4^t \to \infty$$

as $t \to \infty$. So the given series is not absolutely convergent. On the other hand,

$$f(n) > f(n+1), \qquad \lim_{n\to\infty} f(x) = 0.$$

Consequently, by the Alternating Series Test, the series

$$\sum_{n=4}^{\infty} (-1)^n f(n)$$

is convergent, and so the given series is

$$\boxed{\text{conditionally convergent}}.$$

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keywords:

CalC12f09s

55:06, calculus3, multiple choice, < 1 min, wording-variable.

009

Which one of the following properties does the series

$$\sum_{n=1}^{\infty} \frac{2n+3}{(2n)!}$$

have?

1. divergent

2. absolutely convergent **correct**

3. conditionally convergent

Explanation:

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keywords:

CalC12f10s

55:06, calculus3, multiple choice, > 1 min, wording-variable.

010

Which one of the following properties does the series

$$\sum_{n=1}^{\infty} 2^{-n}\, n!$$

have?

1. absolutely convergent

2. conditionally convergent

3. divergent **correct**

Explanation:

The given series can be written in the form

$$\sum_{n=1}^{\infty} 2^{-n}\, n! = \sum_{n=1}^{\infty} a_n$$

with

$$a_n = \frac{n!}{2^n}.$$

But

$$\frac{n!}{2^n} = \frac{1 \cdot 2 \cdot 3 \ldots \cdot n}{2 \cdot 2 \cdot \ldots \cdot 2} = \Big(\frac{1}{2}\Big)\Big(\frac{2}{2}\Big)\ldots\Big(\frac{n}{2}\Big),$$

so

$$a_{n+1} > 2a_n$$

for all $n > 4$. Thus

$$\lim_{n\to\infty} a_n = \infty.$$

Conseqently, the given series

$$\boxed{\text{diverges}}.$$

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keywords:

CalC12f33b
55:06, calculus3, multiple choice, < 1 min, wording-variable.

011

Decide which of the following series converge(s)

$$(A) \quad \sum_{n=1}^{\infty} \frac{n^8}{n+3}\left(\frac{3}{8}\right)^n$$

$$(B) \quad \sum_{n=1}^{\infty} \frac{\sqrt{n}-4}{\sqrt{n}+5}\left(\frac{5}{4}\right)^n$$

$$(C) \quad \sum_{n=1}^{\infty} \left(\frac{4n+5}{n^3+8}\right)^n$$

1. C only

2. A and B

3. B only

4. all of them

5. A and C **correct**

Explanation:

We compute one of

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n}, \quad \lim_{n\to\infty} (a_n)^{1/n}$$

for each of the given series.

(A) The ratio test is the better one to use:

$$\frac{a_{n+1}}{a_n} = \frac{3}{8}\left(\frac{n+1}{n}\right)^8 \frac{n+3}{n+1+3} \longrightarrow \frac{3}{8} < 1$$

as $n \to \infty$, so series (A) converges.

(B) The ratio test is the better one to apply:

$$\frac{a_{n+1}}{a_n} = \frac{5}{4}\left(\frac{(\sqrt{n+1}-4)(\sqrt{n}+5)}{(\sqrt{n}-4)(\sqrt{n+1}+5)}\right) \longrightarrow \frac{5}{4} \text{ as } n \to \infty,$$

so series (B) diverges.

(C) The root test is the better one to apply:

$$(a_n)^{1/n} = \frac{4n+5}{n^3+8} \longrightarrow 0,$$

as $n \to \infty$, so series (C) converges.

Consequently, of the given infinite series,

$$\boxed{\text{only } A \text{ and } C}$$

converge.

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keywords:

CalC12h05s
55:08, calculus3, multiple choice, > 1 min, normal.

012

Find the interval of convergence of the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{4n^2+5}.$$

1. converges only at $x = 0$

2. interval of cgce $= [-1, 1]$ **correct**

3. interval of cgce $= [-1, 1)$

4. interval of cgce $= [-4, 5]$

5. interval of cgce $= (-4, 5]$

6. interval of cgce $= (-1, 1]$

Explanation:

When

$$a_n = (-1)^n \frac{x^n}{4n^2+5},$$

then

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|-\frac{x^{n+1}}{4(n+1)^2+5}\frac{4n^2+5}{x^n}\right| = |x|\left(\frac{4n^2+5}{4(n+1)^2+5}\right).$$

But

$$4(n+1)^2+5 = 4n^2+8n+9,$$

while

$$\lim_{n\to\infty}\frac{4n^2+5}{4n^2+8n+9} = 1.$$

Thus

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = |x|.$$

By the Ratio Test, therefore, the given series

(i) converges when $|x| < 1$,

(ii) diverges when $|x| > 1$.

We have still to check what happens at the endpoints $x = \pm 1$. At $x = -1$ the series becomes

$$(*) \qquad \sum_{n=1}^{\infty}\frac{1}{4n^2+5}.$$

Applying the Integral Test with

$$f(x) = \frac{1}{4x^2+5}$$

we see that f is continuous, positive, and decreasing on $[1, \infty)$; in addition,the improper integral

$$I = \int_1^{\infty} f(x)\,dx$$

converges, so the infinite series $(*)$ converges also.

On the other hand, at $x = 1$, the series becomes

$$(\ddagger) \qquad \sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2+5}.$$

which is an alternating series

$$\sum_{n=1}^{\infty}(-1)^n a_n, \qquad a_n = f(x)$$

with

$$f(x) = \frac{1}{4x^2+5}$$

the same continuous, positive and decreasing function as before. As

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{1}{4x+5} = 0,$$

the Alternating Series Test thus ensures that $(\ddagger)$ too converges.

Consequently, the

$$\boxed{\text{interval of convergence} = [-1, 1]}.$$

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keywords:

CalC12i03c

55:09, calculus3, multiple choice, < 1 min, wording-variable.

013

Find a power series representation for the function

$$f(z) = \frac{1}{z-2}.$$

1. $f(z) = \sum_{n=0}^{\infty}(-1)^{n-1}2^{n+1}z^n$

2. $f(z) = \sum_{n=0}^{\infty}(-1)^n 2^n z^n$

3. $f(z) = -\sum_{n=0}^{\infty}\frac{1}{2^{n+1}}z^n$ **correct**

4. $f(z) = -\sum_{n=0}^{\infty}2^n z^n$

5. $f(z) = \sum_{n=0}^{\infty}\frac{1}{2^{n+1}}z^n$

Explanation:
We know that

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots = \sum_{n=0}^{\infty} x^n .$$

On the other hand,

$$\frac{1}{z-2} = -\frac{1}{2}\Big(\frac{1}{1-(z/2)}\Big).$$

Thus

$$f(z) = -\frac{1}{2}\sum_{n=0}^{\infty}\Big(\frac{z}{2}\Big)^n = -\frac{1}{2}\sum_{n=0}^{\infty}\frac{1}{2^n}z^n .$$

Consequently,

$$\boxed{f(z) = -\sum_{n=0}^{\infty}\frac{1}{2^{n+1}}z^n}$$

with $|z| < 2$.

keywords:

CalC12i21a

55:09, calculus3, multiple choice, > 1 min, wording-variable.

014

Find a power series representation for the function

$$f(y) = \ln\sqrt{\frac{1+4y}{1-4y}} .$$

(*Hint*: remember properties of logs.)

1. $f(y) = \sum_{n=1}^{\infty}\frac{4^{2n-1}}{2n-1}y^{2n-1}$ **correct**

2. $f(y) = \sum_{n=1}^{\infty}\frac{(-1)^n 4^{2n}}{2n-1}y^{2n-1}$

3. $f(y) = \sum_{n=1}^{\infty}\frac{1}{2n-1}y^{2n-1}$

4. $f(y) = \sum_{n=1}^{\infty}\frac{1}{2n}y^{2n}$

5. $f(y) = \sum_{n=1}^{\infty}\frac{4^{2n}}{2n-1}y^{2n}$

Explanation:
We know that

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}x^n ,$$

while

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots = -\sum_{n=1}^{\infty}\frac{1}{n}x^n .$$

Thus

$$\ln(1+x) - \ln(1-x) = 2\Big\{x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\Big\} = 2\Big\{\sum_{n=1}^{\infty}\frac{1}{2n-1}x^{2n-1}\Big\}.$$

Now by properties of logs,

$$\ln\sqrt{\frac{1+4y}{1-4y}} = \frac{1}{2}\ln\frac{1+4y}{1-4y} = \frac{1}{2}\Big\{\ln(1+4y) - \ln(1-4y)\Big\}.$$

Thus

$$f(y) = \frac{2}{2}\Big\{\sum_{n=1}^{\infty}\frac{1}{2n-1}(4y)^{2n-1}\Big\},$$

and so

$$\boxed{f(y) = \sum_{n=1}^{\infty}\frac{4^{2n-1}}{2n-1}y^{2n-1}}.$$

keywords:

CalC12f33a

55:06, calculus3, multiple choice, < 1 min, wording-variable.

015

Determine which, if any, of the following series diverge.

$$(A)\quad \sum_{n=1}^{\infty} \frac{(6n)^n}{n!}$$

$$(B)\quad \sum_{n=1}^{\infty} \frac{2^n}{(n+3)^n}$$

$$(C)\quad \sum_{n=1}^{\infty} \left(\frac{2n}{n+2}\right)^n \left(\frac{2}{3}\right)^n$$

1. B only

2. A and C **correct**

3. all of them

4. A and B

5. A only

6. B and C

7. none of them

8. C only

Explanation:

To check for divergence we shall use either the Ratio test or the Root test which means computing one or other of

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|, \qquad \lim_{n\to\infty} |a_n|^{1/n}$$

for each of the given series.

(A) The ratio test is the better one to use because

$$\left|\frac{a_{n+1}}{a_n}\right| = 6\left(\frac{n!}{(n+1)!}\frac{(n+1)^{n+1}}{n^n}\right).$$

Now

$$\frac{n!}{(n+1)!} = \frac{1}{n+1},$$

while

$$\frac{(n+1)^{n+1}}{n^n} = (n+1)\left(\frac{n+1}{n}\right)^n.$$

Thus

$$\left|\frac{a_{n+1}}{a_n}\right| = 6\left(\frac{n+1}{n}\right)^n \longrightarrow 6e > 1$$

as $n \to \infty$, so series (A) diverges.

(B) The root test is the better one to apply because

$$|a_n|^{1/n} = \frac{2}{n+3} \longrightarrow 0$$

as $n \to \infty$, so series (B) converges.

(C) Again the root test is the better one to apply because of the n^{th} powers. For then

$$|a_n|^{1/n} = \frac{2}{3}\left(\frac{2n}{n+2}\right) \longrightarrow \frac{4}{3} > 1$$

as $n \to \infty$, so series (C) diverges.

Consequently, of the given infinite series,

$$\boxed{\text{only } A \text{ and } C}$$

diverge.

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keywords:

CalC12j11s

55:10, calculus3, multiple choice, > 1 min, wording-variable.

016

Find the Taylor series representation for f centered at $x = 1$ when

$$f(x) = 4 + x - 4x^2.$$

1. $f(x) = 1 - 7(x-1) - 4(x-1)^2$ **correct**

2. $f(x) = 1 + (x-1) + 4(x-1)^2$

3. $f(x) = 4 + (x-1) - 8(x-1)^2$

4. $f(x) = 1 - 7(x-1) - 8(x-1)^2$

5. $f(x) = 4 - 7(x-1) + 8(x-1)^2$

6. $f(x) = 4 + (x-1) - 4(x-1)^2$

Explanation:

For a function f the Taylor series representation centered at $x = 1$ is given by

$$f(x) = \sum_{n=0} \frac{1}{n!} f^{(n)}(1)(x-1)^n .$$

Since f is a polynomial of degree 2, however, $f^{(n)} = 0$ for all $n \geq 3$, so we have only to calculate derivatives of f up to order 2:

$$f'(x) = 1 - 8x, \qquad f''(x) = -8 .$$

Thus

$$f(1) = 1, \quad f'(1) = -7, \quad f''(1) = -8 .$$

Consequently,

$$\boxed{f(x) = 1 - 7(x-1) - 4(x-1)^2} .$$

---

keywords: Taylor series, polynomial function,

CalC12j14a

55:10, calculus3, multiple choice, < 1 min, wording-variable.

017

Find the degree three Taylor polynomial T_3 centered at $x = 0$ for f when

$$f(x) = 2\ln(3 - 2x) .$$

1. $T_3(x) = 2\ln 3 + \frac{4}{3}x - \frac{4}{9}x^2 + \frac{8}{81}x^3$

2. $T_3(x) = \frac{4}{3}x - \frac{4}{9}x^2 + \frac{16}{81}x^3$

3. $T_3(x) = 2\ln 3 - \frac{4}{3}x - \frac{4}{9}x^2 - \frac{16}{81}x^3$ **correct**

4. $T_3(x) = 2\ln 3 - \frac{4}{3}x + \frac{4}{9}x^2 - \frac{16}{81}x^3$

5. $T_3(x) = \ln 3 - \frac{4}{3}x - \frac{4}{9}x^2 - \frac{16}{81}x^3$

Explanation:

The degree three Taylor polynomial centered at $x = 0$ for a function f is defined by

$$p_3(x) = f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 + \frac{1}{3!}f'''(0)x^3 .$$

We use the Chain Rule repeatedly to compute the derivatives of f:

$$f'(x) = -\frac{4}{3-2x}, \quad f''(x) = -\frac{8}{(3-2x)^2},$$

and

$$f'''(x) = -\frac{32}{(3-2x)^3} .$$

Thus

$$f(0) = 2\ln 3, \quad f'(0) = -\frac{4}{3},$$

$$\frac{1}{2!}f''(0) = -\frac{4}{9}, \quad \frac{1}{3!}f'''(0) = -\frac{16}{81},$$

and so

$$\boxed{T_3(x) = 2\ln 3 - \frac{4}{3}x - \frac{4}{9}x^2 - \frac{16}{81}x^3} .$$

---

keywords:

CalC12l31b

55:10, calculus3, multiple choice, < 1 min, normal.

018

The demand x for computers at a price of $\$p$ is given by the Demand-price function

$$p = D(x) = 144\left(64 - x^2\right)^{1/2}$$

where $0 \leq x \leq 8$. Use the degree two Taylor polynomial for D centered at $x = 0$ to calculate an approximate value for the average price $\overline{p}$ (in dollars) over the demand interval $[0, 3]$.

1. $\overline{p} \sim \$1105$

2. $\overline{p} \sim \$1085$

3. $\overline{p} \sim \$1095$

4. $\overline{p} \sim \$1115$

5. $\overline{p} \sim \$1125$ **correct**

Explanation:

The average price $\overline{p}$ over the demand interval $[0, 3]$ is defined by

$$\overline{p} = \frac{1}{3}\int_0^3 D(x)\,dx.$$

When D is approximated by its second degree Taylor polynomial p_2, an approximate value for this average price is thus given by

$$\overline{p} \sim \frac{1}{3}\int_0^3 p_2(x)\,dx.$$

Now calculations show that

$$D(0) = 18 \cdot 64, \quad D'(0) = 0, \quad D''(0) = -18,$$

and so its degree two Taylor polynomial centered at $x = 0$ is

$$p_2(x) = 18\Big(64 - \frac{1}{2}x^2\Big).$$

Thus

$$\overline{p} \sim 3(384 - 9),$$

so the degree two Taylor polynomial gives an approximate value

$$\boxed{\overline{p} \sim \$1125}$$

for the average price over the demand interval $[0, 3]$.

---

keywords:

CalC12a24a

55:01, calculus3, multiple choice, > 1 min, wording-variable.

019

Determine if the sequence $\{a_n\}$ converges, when

$$a_n = 9\cos\left(\frac{6\,n\,\pi + 4}{18\,n + 8}\right),$$

and if it does, find its limit.

1. limit $= 9\cos\frac{1}{2}$

2. sequence does not converge

3. limit $= \frac{9}{2}\sqrt{3}$

4. limit $= \frac{9}{2}$ **correct**

5. limit $= \cos\frac{1}{2}$

6. limit $= 3\,\pi$

Explanation:

After division,

$$\frac{6\,n\,\pi + 4}{18\,n + 8} = \frac{6\,\pi + \frac{4}{n}}{18 + \frac{8}{n}}.$$

But $\frac{4}{n},\ \frac{8}{n} \to 0$ as $n \to \infty$, so

$$\lim_{n\to\infty} \frac{6\,n\,\pi + 4}{18\,n + 8} = \frac{\pi}{3}.$$

Consequently, since $\cos x$ is continuous as a function of x, the sequence $\{a_n\}$ converges and has

$$\boxed{\text{limit} = 9\cos\frac{\pi}{3} = \frac{9}{2}}.$$

---

keywords: sequence, convergence, limit, continuity

CalC12a17c
55:01, calculus3, multiple choice, > 1 min, wording-variable.
020

Determine if the sequence $\{a_n\}$ converges when

$$a_n = \frac{n^{3n}}{(n-2)^{3n}},$$

and if it does, find its limit

1. limit $= e^{-\frac{2}{3}}$
2. limit $= 1$
3. limit $= e^{\frac{2}{3}}$
4. sequence diverges
5. limit $= e^6$ **correct**
6. limit $= e^{-6}$

Explanation:
By the Laws of Exponents,

$$a_n = \left(\frac{n-2}{n}\right)^{-3n} = \left(1-\frac{2}{n}\right)^{-3n}$$
$$= \left[\left(1-\frac{2}{n}\right)^n\right]^{-3}.$$

But

$$\left(1+\frac{x}{n}\right)^n \longrightarrow e^x$$

as $n \to \infty$. Consequently, $\{a_n\}$ converges and has

$$\boxed{\text{limit} = \left(e^{-2}\right)^{-3} = e^6}.$$

keywords: sequence, e, exponentials, limit

CalC12a31s
55:01, calculus3, multiple choice, > 1 min, wording-variable.
021

Determine whether the sequence $\{a_n\}$ converges or diverges when

$$a_n = \frac{2+\cos^2 n}{3+2^n}.$$

1. converges with limit $= \frac{1}{2}$
2. converges with limit $= 2$
3. converges with limit $= \frac{2}{3}$
4. converges with limit $= 0$ **correct**
5. diverges

Explanation:
Since

$$0 \le \cos^2 n \le 1,$$

we see that

$$\frac{2}{3+2^n} \le a_n \le \frac{3}{3+2^n}.$$

But

$$\lim_{n\to\infty} \frac{2}{3+2^n} = 0 = \lim_{n\to\infty} \frac{3}{3+2^n},$$

so the Squeeze Theorem applies and ensures the sequence $\{a_n\}$

$$\boxed{\text{converges with limit} = 0}.$$

keywords:

CalC12b32b
55:02, calculus3, multiple choice, > 1 min, fixed.
022

Find the sum of the infinite series

$$\sum_{k=1}^{\infty} (\cos^2\theta)^k$$

whenever the series converges.

1. sum $= \cot^2\theta$ **correct**

2. sum $= \tan^2\theta$

3. sum $= \sec^2\theta$

4. sum $= \sin^2\theta\cos^2\theta$

5. sum $= \csc^2\theta$

Explanation:
For general θ the series

$$\sum_{k=1}^{\infty} (\cos^2\theta)^k$$

is an infinite geometric series with common ratio $\cos^2\theta$. Since the series starts at $k = 1$, its sum is thus given by

$$\frac{\cos^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}.$$

Consequently

$$\boxed{\text{sum} = \cot^2\theta}.$$

keywords:

CalC12g01exam1
55:06, calculus3, multiple choice, > 1 min, wording-variable.
023

Determine which, if any, of the series

A. $\displaystyle\sum_{m=3}^{\infty} \frac{m+3}{m^2\ln m+2}$

B. $1+\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\dfrac{1}{16}+\dots$

are convergent.

1. neither of them

2. both of them

3. A only

4. B only **correct**

Explanation:

A. Divergent: use Limit Comparison Test and Integral Test with

$$f(x) = \frac{1}{x\ln x}.$$

B. Convergent: given series is a geometric series

$$\sum_{n=0}^{\infty} ar^n$$

with $a = 1$ and $r = \frac{1}{2} < 1$.

keywords:

CalC12g03exam
55:06, calculus3, multiple choice, > 1 min, wording-variable.
024

Determine which, if any, of the series

A. $\displaystyle\sum_{k=1}^{\infty} \frac{k+2}{k\,3^k}$

B. $\displaystyle\sum_{n=2}^{\infty} \frac{3\sqrt{n}}{n\sqrt{n}-2}$

are convergent.

1. A only **correct**

2. both of them

3. neither of them

4. B only

Explanation:

A. Convergent: use Limit Comparison Test and Geometric Series.

B. Divergent: use Limit Comparison Test and p-series Test with $p = 1$.

---

keywords:

CalC12g01a

55:06, calculus3, multiple choice, > 1 min, wording-variable.

025

Which, if any, of the following statements are true?

A. If $\sum a_n$ is divergent, then $\sum |a_n|$ is divergent.

B. If $\sum a_n$ converges, then $\lim\limits_{n \to \infty} a_n = 0$.

C. The Ratio Test can be used to determine whether $\sum 1/n!$ converges.

1. A and B only

2. C only

3. B and C only

4. none of them

5. A and C only

6. all of them **correct**

7. A only

8. B only

Explanation:

A. True: if $\sum |a_n|$ were convergent, then $\sum a_n$ would be absolutely convergent, hence convergent.

B. True. To say that $\sum a_n$ converges is to say that the limit $\lim\limits_{n \to \infty} s_n$ of its partial sums

$$s_n = a_1 + a_2 + \ldots + a_n$$

converges. But then

$$\lim_{n \to \infty} a_n = s_n - s_{n-1} = 0\,.$$

C. True: when $a_n = 1/n!$, then

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{n+1} \longrightarrow 0$$

as $n \to, \infty$, so $\sum a_n$ is convergent by Ratio Test.

---

keywords:

CalC12h30a

55:08, calculus3, multiple choice, > 1 min, wording-variable.

026

If the series

$$\sum_{n=0}^{\infty} c_n x^n$$

converges when $x = -4$ and diverges when $x = 5$, which of the following series converge without further restrictions on $\{c_n\}$?

A. $\displaystyle\sum_{n=0}^{\infty} c_n 8^n$

B. $\displaystyle\sum_{n=0}^{\infty} (-1)^n c_n 9^n$

C. $\displaystyle\sum_{n=0}^{\infty} c_n (-4)^{n+1}$

1. C only **correct**

2. A and B only

3. A only

4. B only

5. B and C only

6. all of them

7. A and C only

8. none of them

Explanation:

A. The series

$$\sum_{n=0}^{\infty} c_n x^n$$

diverges for all $|x| > 5$, hence for $x = 8$.

B. The series

$$\sum_{n=0}^{\infty} c_n x^n$$

diverges for all $|x| > 5$, hence for $x = -9$.

C. Since

$$\sum_{n=0}^{\infty} c_n(-4)^{n+1} = -4\left(\sum_{n=0}^{\infty} c_n(-4)^n\right)$$

the series

$$\sum_{n=0}^{\infty} c_n(-4)^{n+1}$$

converges.

---

keywords:

CalC12j17a

55:10, calculus3, multiple choice, > 1 min, wording-variable.

027

Find the degree three Taylor polynomial T_3 centered at $x = 0$ for f when

$$f(x) = \frac{1}{(4x+1)^{1/2}}.$$

1. $T_3(x) = x - 2x^2 - 20x^3$

2. $T_3(x) = 1 + 2x - 6x^2 + 20x^3$

3. $T_3(x) = 1 - 2x + 12x^2 - 120x^3$

4. $T_3(x) = 1 + 2x - 2x^2 - 20x^3$

5. $T_3(x) = x + 6x^2 + 20x^3$

6. $T_3(x) = 1 - 2x + 6x^2 - 20x^3$ **correct**

Explanation:

The degree three Taylor polynomial centered at $x = 0$ for a function f is defined by

$$T_3(x) = f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 + \frac{1}{3!}f'''(0)x^3.$$

When

$$f(x) = \frac{1}{(4x+1)^{1/2}}$$

we use the Chain Rule repeatedly to compute the derivatives of f:

$$f'(x) = -\frac{4}{2(4x+1)^{3/2}},$$

$$f''(x) = \frac{3\cdot 4^2}{4(4x+1)^{5/2}},$$

and

$$f'''(x) = -\frac{15\cdot 4^3}{8(4x+1)^{7/2}}.$$

Thus

$$f(0) = 1, \qquad f'(0) = -2,$$

while

$$\frac{f''(0)}{2!} = 6, \qquad \frac{f'''(0)}{3!} = -20.$$

Consequently,

$$\boxed{T_3(x) = 1 - 2x + 6x^2 - 20x^3}.$$

---

keywords:

CalC12a17s

55:01, calculus3, multiple choice, < 1 min, wording-variable.

028

Determine whether the sequence $\{a_n\}$ converges or diverges when

$$a_n = \frac{3 - 4n^2}{n + 3n^2},$$

and if it converges, find the limit.

1. converges with limit $= 3$
2. converges with limit $= -\frac{1}{4}$
3. converges with limit $= -\frac{4}{3}$ **correct**
4. diverges
5. converges with limit $= 0$

Explanation:

Since

$$a_n = \frac{3 - 4n^2}{n + 3n^2} = \frac{\frac{3}{n^2} - 4}{\frac{1}{n} + 3},$$

while

$$\lim_{n\to\infty} \frac{3}{n^2} = \lim_{n\to\infty} \frac{1}{n} = 0,$$

properties of limits ensure that $\lim_{n\to\infty} a_n$ exists and that

$$\lim_{n\to\infty} a_n = -\frac{4}{3}.$$

Consequently, the sequence $\{a_n\}$

$$\boxed{\text{converges with limit} = -\frac{4}{3}.}$$

keywords:

CalC12a28s

55:01, calculus3, multiple choice, > 1 min, wording-variable.

029

Determine whether the sequence $\{a_n\}$ converges or diverges when

$$a_n = \frac{\ln(3n^2)}{\ln(5n^3)},$$

and if it converges, find the limit.

1. converges with limit $= 0$
2. converges with limit $= \frac{3}{5}$
3. diverges
4. converges with limit $= \frac{\ln 3}{\ln 5}$
5. converges with limit $= \frac{2}{3}$ **correct**

Explanation:

By properties of logs,

$$\ln(3n^2) = \ln 3 + 2\ln n,$$
$$\ln(5n^3) = \ln 5 + 3\ln n.$$

Thus

$$a_n = \frac{\ln 3 + 2\ln n}{\ln 5 + 3\ln n} = \frac{2 + \frac{\ln 3}{\ln n}}{3 + \frac{\ln 5}{\ln n}}.$$

On the other hand,

$$\lim_{n\to\infty} \frac{\ln 3}{\ln n} = \lim_{n\to\infty} \frac{\ln 5}{\ln n} = 0.$$

Properties of limits thus ensure that the given sequence

$$\boxed{\text{converges with limit} = \frac{2}{3}}.$$

keywords: sequence, log, limit, convergence

CalC12b32f
55:02, calculus3, multiple choice, > 1 min, fixed.

030

Find the sum of the infinite series

$$\tan^2\theta - \tan^4\theta + \tan^6\theta + \ldots + (-1)^{n-1}\tan^{2n}\theta + \ldots$$

whenever the series converges.

1. sum $= \cos^2\theta$
2. sum $= \tan^2\theta$
3. sum $= \sin^2\theta$ **correct**
4. sum $= -\cos^2\theta$
5. sum $= -\sin^2\theta$

Explanation:
For general θ the series

$$\tan^2\theta - \tan^4\theta + \tan^6\theta + \ldots + (-1)^{n-1}\tan^{2n}\theta + \ldots$$

is an infinite geometric series whose common ratio is $-\tan^2\theta$. Since the initial term in this series is $\tan^2\theta$, its sum is thus given by

$$\frac{\tan^2\theta}{1+\tan^2\theta} = \frac{\tan^2\theta}{\sec^2\theta}.$$

Consequently

$$\boxed{\text{sum} = \sin^2\theta}.$$

keywords:

CalC12b49a
55:02, calculus3, multiple choice, > 1 min, wording-variable.

031

Find the n^{th} term, a_n, of an infinite series $\sum_{n=1}^{\infty} a_n$ when the n^{th} partial sum, S_n, of the series is given by

$$S_n = \frac{n}{n+1}.$$

1. $a_n = \dfrac{1}{n(n+1)}$ **correct**
2. $a_n = \dfrac{2}{n}$
3. $a_n = \dfrac{4}{n(n+1)}$
4. $a_n = \dfrac{1}{2n^2}$
5. $a_n = \dfrac{2}{n^2}$
6. $a_n = \dfrac{1}{2n}$

Explanation:
Since $S_n = a_1 + a_2 + \cdots + a_n$, we see that

$$a_1 = S_1, \quad a_n = S_n - S_{n-1} \ (n > 1).$$

But

$$S_n = \frac{n}{n+1} = 1 - \frac{1}{n+1}.$$

Thus $a_1 = \frac{1}{2}$, while

$$a_n = \frac{1}{n} - \frac{1}{n+1}, \quad (n > 1).$$

Consequently,

$$\boxed{a_n = \frac{1}{n} - \frac{1}{n+1} = \frac{1}{n(n+1)}}$$

for all n.

keywords:

CalC12g01exam2
55:06, calculus3, multiple choice, > 1 min, wording-variable.

032

Determine which, if any, of the series

A. $\frac{3}{5} + \frac{4}{6} + \frac{5}{7} + \frac{6}{8} + \frac{7}{9} + \ldots$

B. $\displaystyle\sum_{m=3}^{\infty} \frac{m+2}{(m \ln m)^2}$

are divergent.

1. A only **correct**

2. neither of them

3. B only

4. both of them

Explanation:

A. Divergent by Divergent Series Test: given series is of the form

$$\sum_{n=3}^{\infty} \frac{n}{n+2},$$

and so

$$\lim_{n \to \infty} a_n = 1 \neq 0.$$

B. Convergent: use Limit Comparison Test and Integral Test with

$$f(x) = \frac{1}{x(\ln x)^2}.$$

keywords:

CalC12g01a

55:06, calculus3, multiple choice, > 1 min, wording-variable.

033

Which, if any, of the following statements are true?

A. If $\sum a_n$ is divergent, then $\sum |a_n|$ is divergent.

B. If $\sum a_n$ converges, then $\lim_{n \to \infty} a_n = 0$.

C. The Ratio Test can be used to determine whether $\sum 1/n!$ converges.

1. B only

2. none of them

3. A only

4. A and B only

5. C only

6. all of them **correct**

7. A and C only

8. B and C only

Explanation:

A. True: if $\sum |a_n|$ were convergent, then $\sum a_n$ would be absolutely convergent, hence convergent.

B. True. To say that $\sum a_n$ converges is to say that the limit $\lim_{n \to \infty} s_n$ of its partial sums

$$s_n = a_1 + a_2 + \ldots + a_n$$

converges. But then

$$\lim_{n \to \infty} a_n = s_n - s_{n-1} = 0.$$

C. True: when $a_n = 1/n!$, then

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{n+1} \longrightarrow 0$$

as $n \to, \infty$, so $\sum a_n$ is convergent by Ratio Test.

keywords:

CalC12h10b

55:08, calculus3, multiple choice, > 1 min, wording-variable.

034

Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{8^n}{5n^8+3} x^n .$$

1. interval $= \left[-\frac{8}{5}, \frac{8}{5}\right)$
2. interval $= \left[-\frac{1}{8}, \frac{1}{8}\right)$
3. interval $= \left[-\frac{8}{5}, \frac{8}{5}\right]$
4. interval $= [-8, 8]$
5. interval $= [-8, 8)$
6. interval $= \left[-\frac{1}{8}, \frac{1}{8}\right]$ **correct**

Explanation:

We first apply the Ratio Test to the infinite series

$$\sum_{n=0}^{\infty} \frac{8^n}{5n^8+3} |x|^n .$$

For this series

$$\frac{a_{n+1}}{a_n} = \frac{8^{n+1}}{8^n}\left[\frac{5n^8+3}{5(n+1)^8+3}\right] |x| .$$

But

$$\frac{5n^8+3}{5(n+1)^8+3} = \frac{5+\frac{3}{n^8}}{5\left(\frac{n+1}{n}\right)^8+\frac{3}{n^8}},$$

so

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$$= \lim_{n\to\infty} \frac{8|x|\left(5+\frac{3}{n^8}\right)}{5\left(\frac{n+1}{n}\right)^8+\frac{3}{n^8}} = 8|x| .$$

Thus the given power series will converge on the interval $(-\frac{1}{8}, \frac{1}{8})$.

For convergence at the endpoints $x = \pm\frac{1}{8}$ we have to check which, if any, of the series

$$\sum_{n=0}^{\infty} \frac{1}{5n^8+3}, \quad \sum_{n=0}^{\infty} (-1)^n \frac{1}{5n^8+3}$$

converges. Now the Integral test ensures that the series

$$\sum_{n=0}^{\infty} \frac{1}{n^8},$$

hence by the Comparison test, the series

$$\sum_{n=0}^{\infty} \frac{1}{5n^8+3}$$

converges also. As

$$\sum_{n=0}^{\infty} \left|(-1)^n \frac{1}{5n^8+3}\right| = \sum_{n=0}^{\infty} \frac{1}{5n^8+3}$$

the power series thus converges at $x = \pm\frac{1}{8}$. Consequently,

$$\boxed{\text{interval} = \left[-\frac{1}{8}, \frac{1}{8}\right]}.$$

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keywords: power series, interval of convergence, ratio test, integral test, comparison test

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CalC12h30a

55:08, calculus3, multiple choice, > 1 min, wording-variable.

035

If the series

$$\sum_{n=0}^{\infty} c_n x^n$$

converges when $x = -4$ and diverges when $x = 5$, which of the following series converge without further restrictions on $\{c_n\}$?

A. $\sum_{n=0}^{\infty} c_n 8^n$

B. $\sum_{n=0}^{\infty} (-1)^n c_n 9^n$

C. $\sum_{n=0}^{\infty} c_n(-4)^{n+1}$

1. C only **correct**

2. A and C only

3. A and B only

4. none of them

5. B only

6. B and C only

7. A only

8. all of them

Explanation:

A. The series

$$\sum_{n=0}^{\infty} c_n x^n$$

diverges for all $|x| > 5$, hence for $x = 8$.

B. The series

$$\sum_{n=0}^{\infty} c_n x^n$$

diverges for all $|x| > 5$, hence for $x = -9$.

C. Since

$$\sum_{n=0}^{\infty} c_n(-4)^{n+1} = -4\left(\sum_{n=0}^{\infty} c_n(-4)^n\right)$$

the series

$$\sum_{n=0}^{\infty} c_n(-4)^{n+1}$$

converges.

---

keywords:

CalC12i03c

55:09, calculus3, multiple choice, < 1 min, wording-variable.

036

Find a power series representation for the function

$$f(z) = \frac{1}{z-2}.$$

1. $f(z) = \sum_{n=0}^{\infty} (-1)^{n-1} 2^{n+1} z^n$

2. $f(z) = \sum_{n=0}^{\infty} (-1)^n 2^n z^n$

3. $f(z) = -\sum_{n=0}^{\infty} \frac{1}{2^{n+1}} z^n$ **correct**

4. $f(z) = \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} z^n$

5. $f(z) = -\sum_{n=0}^{\infty} 2^n z^n$

Explanation:

We know that

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots = \sum_{n=0}^{\infty} x^n.$$

On the other hand,

$$\frac{1}{z-2} = -\frac{1}{2}\left(\frac{1}{1-(z/2)}\right).$$

Thus

$$f(z) = -\frac{1}{2}\sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n = -\frac{1}{2}\sum_{n=0}^{\infty} \frac{1}{2^n} z^n.$$

Consequently,

$$\boxed{f(z) = -\sum_{n=0}^{\infty} \frac{1}{2^{n+1}} z^n}$$

with $|z| < 2$.

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keywords:

CalC12i22a

55:09, calculus3, multiple choice, > 1 min, wording-variable.

037

Find a power series representation for the function

$$f(z) = \ln\sqrt{\frac{1-2z}{1+2z}}.$$

(*Hint*: remember properties of logs.)

1. $f(z) = -\sum_{n=1}^{\infty} \frac{2^{2n-1}}{2n-1} z^{2n-1}$ **correct**
2. $f(z) = \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n-1}}{2n-1} z^{2n-1}$
3. $f(z) = \sum_{n=1}^{\infty} \frac{2^{2n-1}}{2n-1} z^{2n-1}$
4. $f(z) = -\sum_{n=1}^{\infty} \frac{2^{2n}}{2n} z^{2n}$
5. $f(z) = \sum_{n=1}^{\infty} \frac{2^{2n}}{2n} z^{2n}$

Explanation:

We know that

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots = \sum_{n=1} \frac{(-1)^{n-1}}{n} x^n,$$

while

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \dots = -\sum_{n=1} \frac{1}{n} x^n.$$

Thus

$$\ln(1-x) - \ln(1+x) = -2\left\{x + \frac{x^3}{3} + \frac{x^5}{5} + \dots\right\} = -2\left\{\sum_{n=1}^{\infty} \frac{1}{2n-1} x^{2n-1}\right\}.$$

Now by properties of logs,

$$\ln\sqrt{\frac{1-2z}{1+2z}} = \frac{1}{2}\ln\frac{1-2z}{1+2z} = \frac{1}{2}\left\{\ln(1-2z) - \ln(1+2z)\right\}.$$

Thus

$$f(z) = -\frac{2}{2}\left\{\sum_{n=1}^{\infty} \frac{1}{2n-1}(2z)^{2n-1}\right\},$$

and so

$$\boxed{f(z) = -\sum_{n=1}^{\infty} \frac{2^{2n-1}}{2n-1} z^{2n-1}}.$$

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keywords:

CalC12j17a

55:10, calculus3, multiple choice, > 1 min, wording-variable.

038

Find the degree three Taylor polynomial T_3 centered at $x = 0$ for f when

$$f(x) = \frac{1}{(4x+1)^{1/2}}.$$

1. $T_3(x) = x - 2x^2 - 20x^3$
2. $T_3(x) = 1 + 2x - 2x^2 - 20x^3$
3. $T_3(x) = 1 + 2x - 6x^2 + 20x^3$
4. $T_3(x) = x + 6x^2 + 20x^3$
5. $T_3(x) = 1 - 2x + 12x^2 - 120x^3$
6. $T_3(x) = 1 - 2x + 6x^2 - 20x^3$ **correct**

Explanation:

The degree three Taylor polynomial centered at $x = 0$ for a function f is defined by

$$T_3(x) = f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 + \frac{1}{3!}f'''(0)x^3 .$$

When

$$f(x) = \frac{1}{(4x+1)^{1/2}}$$

we use the Chain Rule repeatedly to compute the derivatives of f:

$$f'(x) = -\frac{4}{2(4x+1)^{3/2}},$$

$$f''(x) = \frac{3\cdot 4^2}{4(4x+1)^{5/2}},$$

and

$$f'''(x) = -\frac{15\cdot 4^3}{8(4x+1)^{7/2}}.$$

Thus

$$f(0) = 1, \qquad f'(0) = -2,$$

while

$$\frac{f''(0)}{2!} = 6, \qquad \frac{f'''(0)}{3!} = -20 .$$

Consequently,

$$\boxed{T_3(x) = 1 - 2x + 6x^2 - 20x^3}.$$

keywords:

CalC12b49a

55:02, calculus3, multiple choice, > 1 min, wording-variable.

039

Find the n^{th} term, a_n, of an infinite series $\sum_{n=1}^{\infty} a_n$ when the n^{th} partial sum, S_n, of the series is given by

$$S_n = \frac{n}{n+1}.$$

1. $a_n = \dfrac{1}{n(n+1)}$ **correct**
2. $a_n = \dfrac{2}{n}$
3. $a_n = \dfrac{2}{n^2}$
4. $a_n = \dfrac{1}{2n}$
5. $a_n = \dfrac{1}{2n^2}$
6. $a_n = \dfrac{4}{n(n+1)}$

Explanation:

Since $S_n = a_1 + a_2 + \cdots + a_n$, we see that

$$a_1 = S_1, \quad a_n = S_n - S_{n-1} \ (n > 1).$$

But

$$S_n = \frac{n}{n+1} = 1 - \frac{1}{n+1}.$$

Thus $a_1 = \frac{1}{2}$, while

$$a_n = \frac{1}{n} - \frac{1}{n+1}, \quad (n > 1).$$

Consequently,

$$\boxed{a_n = \frac{1}{n} - \frac{1}{n+1} = \frac{1}{n(n+1)}}$$

for all n.

keywords:

CalC12h05s

55:08, calculus3, multiple choice, > 1 min, normal.

040

Find the interval of convergence of the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{4n^2+5}.$$

1. interval of cgce $= (-4, 5]$

2. interval of cgce $= [-1, 1]$ **correct**

3. interval of cgce $= [-1, 1)$

4. interval of cgce $= (-1, 1]$

5. interval of cgce $= [-4, 5]$

6. converges only at $x = 0$

Explanation:

When

$$a_n = (-1)^n \frac{x^n}{4n^2+5},$$

then

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|-\frac{x^{n+1}}{4(n+1)^2+5}\frac{4n^2+5}{x^n}\right|$$

$$= |x|\left(\frac{4n^2+5}{4(n+1)^2+5}\right).$$

But

$$4(n+1)^2+5 = 4n^2+8n+9,$$

while

$$\lim_{n\to\infty} \frac{4n^2+5}{4n^2+8n+9} = 1.$$

Thus

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = |x|.$$

By the Ratio Test, therefore, the given series

(i) converges when $|x| < 1$,

(ii) diverges when $|x| > 1$.

We have still to check what happens at the endpoints $x = \pm 1$. At $x = -1$ the series becomes

$$(*) \qquad \sum_{n=1}^{\infty} \frac{1}{4n^2+5}.$$

Applying the Integral Test with

$$f(x) = \frac{1}{4x^2+5}$$

we see that f is continuous, positive, and decreasing on $[1, \infty)$; in addition,the improper integral

$$I = \int_1^{\infty} f(x)\,dx$$

converges, so the infinite series $(*)$ converges also.

On the other hand, at $x = 1$, the series becomes

$$(\ddagger) \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{4n^2+5}.$$

which is an alternating series

$$\sum_{n=1}^{\infty} (-1)^n a_n, \qquad a_n = f(x)$$

with

$$f(x) = \frac{1}{4x^2+5}$$

the same continuous, positive and decreasing function as before. As

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1}{4x+5} = 0,$$

the Alternating Series Test thus ensures that $(\ddagger)$ too converges.

Consequently, the

$$\boxed{\text{interval of convergence} = [-1, 1]}.$$

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keywords: