This print-out should have 18 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering. V1:1, V2:1, V3:3, V4:1, V5:1.

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001 (part 1 of 1) 10 points

Determine whether the series

$$\sum_{n=2}^{\infty} (-1)^n \frac{n}{4\ln n}$$

is conditionally convergent, absolutely convergent, or divergent.

1. series is conditionally convergent

2. series is divergent **correct**

3. series is absolutely convergent

Explanation:

By the Divergence Test, a series

$$\sum_{n=N}^{\infty} (-1)^n a_n$$

will be divergent for each fixed choice of N if

$$\lim_{n\to\infty} a_n \neq 0$$

since it is only the behaviour of a_n as $n \to \infty$ that's important. Now, for the given series, $N = 2$ and

$$a_n = \frac{n}{4\ln n}.$$

But by L'Hospital's Rule,

$$\lim_{x\to\infty} \frac{x}{\ln x} = \lim_{x\to\infty} \frac{1}{1/x} = \infty.$$

Consequently, by the Divergence Test, the given series is

$$\boxed{\text{divergent}}.$$

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keywords:

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002 (part 1 of 1) 10 points

Which one of the following properties does the series

$$\sum_{n=3}^{\infty} (-1)^n \frac{4^n}{(\ln n)^n}$$

have?

1. divergent

2. conditionally convergent

3. absolutely convergent **correct**

Explanation:

The given series can be written in the form

$$\sum_{n=3}^{\infty} (-1)^n \frac{4^n}{(\ln n)^n} = \sum_{n=3}^{\infty} (-1)^n a_n$$

with

$$a_n = \frac{4^n}{(\ln n)^n} > 0.$$

Now

$$\begin{aligned} 0 < \frac{a_{n+1}}{a_n} &= \frac{4(\ln n)^n}{(\ln(n+1))^{n+1}} \\ &= 4\Big(\frac{\ln n}{\ln(n+1)}\Big)^n \Big\{\frac{1}{\ln(n+1)}\Big\} \\ &< \frac{4}{\ln(n+1)}. \end{aligned}$$

Consequently,

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0.$$

In view of the Ratio Test, therefore, the series

$$\sum_{n=3}^{\infty} \left|(-1)^n \frac{4^n}{(\ln n)^n}\right|$$

converges, so the given series is

$$\boxed{\text{absolutely convergent}}.$$

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keywords:

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003 (part 1 of 1) 10 points

Which one of the following properties does the series

$$\sum_{n=1}^{\infty} \frac{(-4)^n}{n!}$$

have?

1. conditionally convergent

2. divergent

3. absolutely convergent **correct**

Explanation:

The given series has the form

$$\sum_{n=1}^{\infty} a_n\,, \qquad a_n = \frac{(-4)^n}{n!}\,.$$

But then

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{4(n!)}{(n+1)!} = \frac{4}{n+1}\,,$$

in which case

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0 < 1\,.$$

Consequently, by the Ratio test, the given series is

$$\boxed{\text{absolutely convergent}}\,.$$

keywords: alternating series, absolutely convergent, divergent, conditionally convergent, Ratio Test

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004 (part 1 of 1) 10 points

Which one of the following properties does the series

$$\sum_{m=3}^{\infty} (-1)^{m-1}\frac{m-2}{m^2+m-4}$$

have?

1. conditionally convergent **correct**

2. absolutely convergent

3. divergent

Explanation:

The given series has the form

$$\sum_{m=3}^{\infty} (-1)^{m-1}\frac{m-1}{m^2+m-4}$$

$$= \sum_{m=3}^{\infty} (-1)^{m-1} f(m)$$

where f is defined by

$$f(x) = \frac{x-2}{x^2+x-4}\,.$$

Notice that $x^2+x-4>0$ on $[3, \infty)$, so the terms in the given series are defined for all $m \ge 3$. On the other hand, $x-2>0$ on $(2, \infty)$, so

$$x > 2 \implies f(x) > 0\,.$$

Now, by the Quotient Rule,

$$f'(x) = \frac{(x^2+x-4)-(x-2)(2x+1)}{(x^2+x-4)^2}$$

$$= -\frac{x^2-4x+2}{(x^2+x-4)^2}\,;$$

in particular, f is decreasing on $[6, \infty)$. Thus by the Limit Comparison Test and the p-series Test with $p=1$, we see that the series

$$\sum_{m=6}^{\infty} f(m)$$

diverges, so the given series fails to be absolutely convergent. But

$$m \ge 6 \implies f(m) > f(m+1)\,,$$

while

$$\lim_{x \to \infty} f(x) = 0.$$

Consequently, by The Alternating Series Test, the given series is

$$\boxed{\text{conditionally convergent}}.$$

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keywords:

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005 (part 1 of 1) 10 points

Determine whether the series

$$3 - 4 + \frac{16}{3} - \frac{64}{9} + \frac{256}{27} + \cdots$$

is convergent or divergent, and if convergent, find its sum.

1. convergent with sum $= 3$

2. convergent with sum $= 4$

3. convergent with sum $= \dfrac{9}{7}$

4. series is divergent **correct**

5. convergent with sum $= \dfrac{8}{7}$

Explanation:

The infinite series

$$3 - 4 + \frac{16}{3} - \frac{64}{9} + \frac{256}{27} \cdots = \sum_{n=1}^{\infty} a\, r^{n-1}$$

is an infinite geometric series with

$$a = 3, \qquad r = -\frac{4}{3}.$$

But an infinite geometric series $\sum_{n=1}^{\infty} a\, r^{n-1}$

(i) converges when $|r| < 1$ and has

$$\text{sum} = \frac{a}{1-r}$$

while it

(ii) diverges when $|r| \geq 1$.

Consequently, the given

$$\boxed{\text{series is divergent}}.$$

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keywords: infinite series, geometric series, divergent

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006 (part 1 of 1) 10 points

Determine the convergence or divergence of the series

$$(A) \qquad \sum_{m=1}^{\infty} \frac{5\ln(3m)}{m^2},$$

and

$$(B) \qquad \sum_{m=1}^{\infty} \frac{\sin^2 m}{m^2+4}.$$

1. both series converge **correct**

2. both series diverge

3. A converges, B diverges

4. A diverges, B converges

Explanation:

(A) The function

$$f(x) = \frac{5\ln 3x}{x^2}$$

is continous and positive on $[\frac{2}{3}, \infty)$; in addition, since

$$f'(x) = 5\left(\frac{1-2\ln 3x}{x^3}\right) < 0$$

on $[\frac{2}{3}, \infty)$, f is also decreasing on this interval. This suggests applying the Integral Test. Now, after Integration by Parts, we see that

$$\int_1^t f(x)\, dx = 5\Big[-\frac{\ln(3x)}{x} - \frac{1}{x}\Big]_1^t,$$

and so

$$\int_1^{\infty} f(x)\,dx \;=\; 5(1+\ln 3)\,.$$

The Integral Test thus ensures that series (A)

$$\boxed{\text{converges}}\,.$$

(B) Note first that the inequalities

$$0 \;<\; \frac{\sin^2 m}{m^2+4} \;\le\; \frac{1}{m^2+4} \;\le\; \frac{1}{m^2}$$

hold for all $n \ge 1$. On the other hand, by the p-series test the series

$$\sum_{m\,=\,1}^{\infty} \frac{1}{m^2}$$

is convergent since $p = 2 > 1$. Thus, by the comparison test, series (B)

$$\boxed{\text{converges}}\,.$$

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keywords:

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007 (part 1 of 1) 10 points

Determine the convergence or divergence of the series

(A) $\quad 1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \ldots\,,$

and

(B) $$\sum_{m\,=\,1}^{\infty} m^3 e^{-m^4}\,.$$

1. both series convergent **correct**

2. both series divergent

3. A divergent, B convergent

4. A convergent, B divergent

Explanation:

(A) The given series has the form

$$1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n^3}\,.$$

This is a p-series with $p = 3 > 1$, so the series converges.

(B) The given series has the form

$$\sum_{m\,=\,1}^{\infty} f(m)$$

with f defined by

$$f(x) \;=\; x^3 e^{-x^4}\,.$$

Note first that f is continuous and positive on $[1, \infty)$; in addition, since

$$f'(x) \;=\; e^{-x^4}(3x^2 - 4x^6) < 0$$

for $x > 1$, f is decreasing on $[1, \infty)$. Thus we can use the Integral Test. Now, by substitution,

$$\int_1^t x^3 e^{-x^4} dx \;=\; \Big[-\frac{1}{4}e^{-x^4}\Big]_1^t\,,$$

and so

$$\int_1^{\infty} x^3 e^{-x^4} dx \;=\; \frac{1}{4e}\,.$$

Since the integral converges, the series converges. This could also be established using the Ratio Test.

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keywords:

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008 (part 1 of 1) 10 points

Which, if any, of the following series converge?

(A) $$\sum_{k\,=\,1}^{\infty} \frac{1}{k\ln k + 3}$$

(B) $$\sum_{n\,=\,5}^{\infty} \Big(\frac{2}{3}\Big)^n$$

1. A and B

2. A but not B

3. B but not A **correct**

4. neither A nor B

Explanation:

(A) Since

$$\lim_{k\to\infty} \frac{k\ln k}{k\ln k+3} = 1,$$

the series

$$\sum_{k=2}^{\infty} \frac{1}{k\ln k+3}$$

converges if and only if the series

$$\sum_{k=2}^{\infty} \frac{1}{k\ln k}$$

converges. But by the Integral Test with

$$f(x) = \frac{1}{x\ln x},$$

this last series diverges, so the given series

$$\boxed{\text{diverges}}\,.$$

(B) Since

$$\sum_{n=3}^{\infty} \left(\frac{2}{3}\right)^n$$

is a geometric series with common ratio $r = \frac{2}{3} < 1$, the series

$$\boxed{\text{converges}}\,.$$

keywords:

009 (part 1 of 1) 10 points

Decide which, if any, of the following series converge.

$$(A)\quad \sum_{n=1}^{\infty} \frac{n^8}{n+3}\left(\frac{3}{8}\right)^n$$

$$(B)\quad \sum_{n=1}^{\infty} \left(\frac{4n+7}{n^3+8}\right)^n$$

1. B only

2. neither of them

3. both of them **correct**

4. A only

Explanation:

We compute one of

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n}, \quad \lim_{n\to\infty} (a_n)^{1/n}$$

for each of the given series.

(A) The ratio test is the better one to use:

$$\frac{a_{n+1}}{a_n} = \frac{3}{8}\left(\frac{n+1}{n}\right)^8 \frac{n+3}{n+1+3} \longrightarrow \frac{3}{8} < 1$$

as $n \to \infty$, so series (A) converges.

(B) The root test is the better one to apply:

$$(a_n)^{1/n} = \frac{4n+7}{n^3+8} \longrightarrow 0,$$

as $n \to \infty$, so series (B) converges also.

Consequently, of the given infinite series,

$$\boxed{\text{both } A \text{ and } B \text{ converge}}$$

converge.

keywords:

010 (part 1 of 1) 10 points

If the improper integral

$$\int_1^{\infty} \frac{1}{x^p}\,dx$$

converges, which of the following statements is (are) always true?

$$(A)\quad \sum_n \frac{1}{n^p} \text{ converges;}$$

(B) $\sum_n \frac{1}{n^{p+1}}$ diverges;

(C) $\sum_n \frac{1}{n^{p-1}}$ converges;

(D) $\sum_n \frac{1}{n^{p-1}}$ diverges;

(E) $\sum_n \frac{1}{n^{p+1}}$ converges.

1. A and E only **correct**

2. B and D only

3. A, D and E

4. A, C and E only

5. A only

Explanation:

To apply the Integral test we need to start with a function f which is positive, continuous and decreasing on $[1, \infty)$. Then the integral test says that the improper integral

$$\int_1^\infty f(x)\, dx$$

converges if and only if the infinite series

$$\sum_{n=1}^\infty f(n)$$

converges.

In the given example

$$f(x) = \frac{1}{x^p},$$

which is a function both continuous and positive on $[1, \infty)$. It will also be decreasing on $[1, \infty)$ if $f'(x) < 0$ for all $x > 1$. But

$$f'(x) = -\frac{p}{x^{p+1}},$$

so f will be decreasing provided $p > 0$. On the other hand, the improper integral

$$\int_1^\infty \frac{1}{x^p}\, dx$$

converges if and only if

$$\lim_{n\to\infty} \int_1^n \frac{1}{x^p}\, dx\,.$$

exists. But

$$\int_1^n \frac{1}{x^p}\, dx = \left[-\frac{1}{px^{p-1}}\right]_1^n = \frac{1}{p}\left(1 - \frac{1}{n^{p-1}}\right).$$

Consequently, the improper integral

$$\int_1^\infty \frac{1}{x^p}\, dx$$

converges if and only if $p > 1$. Hence by the Integral test, the infinite series

$$\sum_{n=1}^\infty \frac{1}{x^p}$$

converges if and only if $p > 1$.

Now we can check which of the statements is (are) always true.

(A) This is always true because of the Integral test.

(B), (E) Since $p > 1 \implies p + 1 > 1$, the Integral test ensures that $\sum_n \frac{1}{n^{p+1}}$ converges. Thus (B) is false and (E) is true.

(C), (D) The series $\sum_n \frac{1}{n^{p-1}}$ converges if and only if $p - 1 > 1$, *i.e.*, when $p > 2$. Since the convergence of the improper integral

$$\int_1^\infty \frac{1}{x^p}\, dx$$

guarantees only that $p > 1$, we see that statements (C) and (D) are true for some values of p and false for others.

Consequently, of the statements,

$$\boxed{\text{only } A \text{ and } E}$$

are always true.

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keywords:

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011 (part 1 of 1) 10 points

Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{4^n}{n!}\, x^n .$$

1. interval $= (-4, 4)$
2. interval $= \left[-\frac{1}{4}, \infty\right)$
3. interval $= (-\infty, \infty)$ **correct**
4. interval $= \left(-\infty, \frac{1}{4}\right)$
5. interval $= \left[-\frac{1}{4}, \frac{1}{4}\right]$
6. interval $= \left(-\frac{1}{4}, \frac{1}{4}\right)$
7. interval $= [-4, 4]$

Explanation:

We apply the ratio test to the infinite series

$$\sum_{n=0}^{\infty} \frac{4^n}{n!}\, |x|^n .$$

For this series,

$$\frac{a_{n+1}}{a_n} = \frac{4^{n+1} n!}{4^n (n+1)!}\, |x| = \frac{4|x|}{n+1} \longrightarrow 0$$

as $n \to \infty$. Thus the given power series converges for all x and so

$$\boxed{\text{interval} = (-\infty, \infty)} .$$

keywords:

012 (part 1 of 1) 10 points

Determine the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n\, 4^n}\, (x+2)^n .$$

1. converges only at $x = -2$
2. interval convergence $= [-6, 2)$
3. interval convergence $= [-2, 6]$
4. interval convergence $= (-6, 2]$ **correct**
5. interval convergence $= (-\infty, \infty)$
6. interval convergence $= [-2, 6)$
7. interval convergence $= (-6, 2)$

Explanation:

The given series has the form

$$\sum_{n=1} a_n (x+2)^n , \quad a_n = \frac{(-1)^n}{n\, 4^n} .$$

Now

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{n}{4(n+1)} = \frac{1}{4} .$$

By the Ratio Test, therefore, the given series

(i) converges when $|x+2| < 4$, and

(ii) diverges when $|x+2| > 4$.

On the other hand, at the points $x+2 = -4$ and $x+2 = 4$ the series reduces to

$$\sum_{n=1}^{\infty} \frac{1}{n}, \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

respectively. But by the p-series Test with $p = 1$, the first of these series diverges; while by the Alternating Series Test the second is convergent. Consequently,

$$\boxed{\text{interval convergence} = (-6, 2]} .$$

keywords:

013 (part 1 of 1) 10 points

Suppose

$$T_4(x) = 6 - 3(x-1) + 7(x-1)^2 - 8(x-1)^3 + 4(x-1)^4$$

is the degree 4 Taylor polynomial centered at $x = 1$ for some function f.

What is the value of $f^{(3)}(1)$?

1. $f^{(3)}(1) = -\frac{8}{3}$

2. $f^{(3)}(1) = -48$ **correct**

3. $f^{(3)}(1) = \frac{8}{3}$

4. $f^{(3)}(1) = -8$

5. $f^{(3)}(1) = 48$

6. $f^{(3)}(1) = 8$

Explanation:

Since

$$T_4(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f^{(3)}(1)}{3!}(x-1)^3 + \frac{f^{(4)}(1)}{4!}(x-1)^4,$$

we see that

$$\boxed{f^{(3)}(1) = -3! \times 8 = -48}.$$

keywords:

014 (part 1 of 1) 10 points

Find the degree 3 Taylor polynomial $T_3(x)$ for f centered at the origin when

$$f(x) = xe^{-3x}.$$

1. $T_3(x) = x - 3x^2 + \frac{9}{2}x^3$ **correct**

2. $T_3(x) = 1 + x - 3x^2 - \frac{9}{2}x^3$

3. $T_3(x) = 1 + x + 3x^2 - \frac{9}{2}x^3$

4. $T_3(x) = 1 + x - 3x^2 + \frac{9}{2}x^3$

5. $T_3(x) = x + 3x^2 - \frac{9}{2}x^3$

6. $T_3(x) = x - 3x^2 - \frac{9}{2}x^3$

Explanation:

The degree 3 Taylor polynomial centered at the origin for a general f is given by

$$T_3(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}\,x^2 + \frac{f'''(0)}{3!}\,x^3.$$

Now when $f(x) = xe^{-3x}$,

$$f'(x) = e^{-3x} - 3xe^{-3x},$$

$$f''(x) = -6e^{-3x} + 9xe^{-3x},$$

while

$$f'''(x) = 27e^{-3x} - 27xe^{-3x}.$$

At $x = 0$, therefore,

$$f(0) = 0, \qquad f'(0) = 1,$$

while

$$\frac{f''(0)}{2!} = -3, \qquad \frac{f'''(0)}{3!} = \frac{9}{2}.$$

Consequently,

$$\boxed{T_3(x) = x - 3x^2 + \frac{9}{2}x^3}.$$

keywords:

015 (part 1 of 1) 10 points

Use the degree 2 Taylor polynomial centered at the origin for f to estimate the definite integral

$$I = \int_0^1 f(x)\,dx$$

when

$$f(x) = \sqrt{1+x^2}.$$

1. $I \approx \dfrac{3}{2}$

2. $I \approx 1$

3. $I \approx \dfrac{7}{6}$ **correct**

4. $I \approx \dfrac{5}{3}$

5. $I \approx \dfrac{4}{3}$

Explanation:
When

$$f(x) = \sqrt{1+x^2} = (1+x^2)^{1/2},$$

we see that

$$f'(x) = x(1+x^2)^{-1/2},$$

while

$$f''(x) = (1+x^2)^{-1/2} - x^2(1+x^2)^{-3/2}.$$

In this case,

$$f(0) = 1, \quad f'(0) = 0, \quad f''(0) = 1.$$

Thus the degree 2 Taylor polynomial for f centered at the origin is

$$T_2(x) = 1 + \frac{1}{2}x^2.$$

But then

$$I \approx \int_0^1 T_2(x)\,dx = \int_0^1 \left(+\frac{1}{2}x^2\right)dx.$$

Consequently,

$$\boxed{I \approx \left[x + \frac{1}{6}x^3\right]_0^1 = \frac{7}{6}}.$$

keywords:

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016 (part 1 of 1) 10 points

Find a power series representation for the function

$$f(z) = \frac{1}{z-3}.$$

1. $f(z) = \displaystyle\sum_{n=0}^{\infty} (-1)^{n-1}3^{n+1}\,z^n$

2. $f(z) = \displaystyle\sum_{n=0}^{\infty} \frac{1}{3^{n+1}}\,z^n$

3. $f(z) = \displaystyle\sum_{n=0}^{\infty} (-1)^n 3^n\,z^n$

4. $f(z) = -\displaystyle\sum_{n=0}^{\infty} \frac{1}{3^{n+1}}\,z^n$ **correct**

5. $f(z) = -\displaystyle\sum_{n=0}^{\infty} 3^n\,z^n$

Explanation:
We know that

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots = \sum_{n=0}^{\infty} x^n.$$

On the other hand,

$$\frac{1}{z-3} = -\frac{1}{3}\left(\frac{1}{1-(z/3)}\right).$$

Thus

$$f(z) = -\frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{z}{3}\right)^n = -\frac{1}{3}\sum_{n=0}^{\infty}\frac{1}{3^n}\,z^n.$$

Consequently,

$$\boxed{f(z) = -\sum_{n=0}^{\infty}\frac{1}{3^{n+1}}\,z^n}$$

with $|z| < 3$.

keywords:

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017 (part 1 of 1) 10 points

Find a power series representation centered at the origin for the function

$$f(x) = \frac{1}{(5-x)^2}.$$

1. $f(x) = \sum_{n=0}^{\infty} \frac{n+1}{5^n} x^n$

2. $f(x) = \sum_{n=1}^{\infty} \frac{1}{5^{n+1}} x^n$

3. $f(x) = \sum_{n=0}^{\infty} \frac{1}{5^{n+1}} x^n$

4. $f(x) = \sum_{n=1}^{\infty} \frac{n}{5^{n+1}} x^{n-1}$ **correct**

5. $f(x) = \sum_{n=1}^{\infty} \frac{n}{5^n} x^{n-1}$

6. $f(x) = \sum_{n=0}^{\infty} (n+1)x^n$

Explanation:

By the known result for geometric series,

$$\frac{1}{5-x} = \frac{1}{5\left(1-\dfrac{x}{5}\right)}$$

$$= \frac{1}{5}\sum_{n=0}^{\infty} \left(\frac{x}{5}\right)^n = \sum_{n=0}^{\infty} \frac{1}{5^{n+1}} x^n.$$

This series converges on $(-5, 5)$.

On the other hand,

$$\frac{1}{(5-x)^2} = \frac{d}{dx}\left(\frac{1}{5-x}\right),$$

and so on $(-5, 5)$,

$$\frac{1}{(5-x)^2} = \frac{d}{dx}\left(\sum_{n=0}^{\infty} \frac{x^n}{5^{n+1}}\right)$$

$$= \sum_{n=1}^{\infty} \frac{n}{5^{n+1}} x^{n-1}.$$

Consequently,

$$\boxed{f(x) = \sum_{n=1}^{\infty} \frac{n}{5^{n+1}} x^{n-1}}.$$

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keywords:

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018 (part 1 of 1) 10 points

Find the Taylor series centered at the origin for the function

$$f(x) = x\cos(3x).$$

1. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{(2n)!} x^{2n+1}$ **correct**

2. $f(x) = \sum_{n=0}^{\infty} \frac{3^n}{n!} x^{n+1}$

3. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} x^{n+1}$

4. $f(x) = \sum_{n=0}^{\infty} \frac{3^{2n}}{(2n)!} x^{2n+1}$

5. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n+1}$

Explanation:

The Taylor series centered at the origin for $\cos x$ is

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}.$$

But then

$$x\cos(3x) = x\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (3x)^{2n}.$$

Consequently, the Taylor series representation for f centered at the origin is

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{(2n)!} x^{2n+1}}.$$

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keywords: