

Practice Problems 7.2*

1. Expand the quantity $\ln \frac{x^3 \cdot y}{z^2}$
2. Express $[\ln x + a \ln y - b \ln z]$ as a single logarithm.
3. Differentiate $F(t) = \ln \frac{(2t+1)^3}{(3t-1)^4}$
4. Differentiate $y = \ln \left(\frac{x+1}{x-1} \right)^{3/5}$
5. Differentiate $y = \ln(\ln x)$

Practice Problems 7.4

1. Find y' and y'' when $y = \log_{10} x$
2. Differentiate $y = \ln(e^{-x} + xe^{-x})$
3. Use logarithmic differentiation to find the derivative of $y = (2x+1)^5(x^4-3)^6$
4. Evaluate the integral $\int_1^e \frac{x^2+x+1}{x} dx$
5. Explain why the natural logarithmic function $y = \ln x$ is used much more frequently in calculus than the other logarithmic functions $y = \log_a x$

Answers to 7.2* Problems

1. $3\ln x + \ln y - 2\ln z$

2. $\ln\left(\frac{x \cdot y^a}{z^b}\right)$

3. $F'(t) = \frac{6}{2t+1} - \frac{12}{3t-1}$

4. $y' = -\frac{6}{5(x^2-1)}$

5. $y' = \frac{1}{x \ln x}$ $y'' = -\frac{1+\ln x}{(x \ln x)^2}$

Answers to 7.4 Problems

1. $y' = \frac{1}{x \ln 10}$ $y'' = -\frac{1}{x^2 \ln 10}$

2. $y' = \frac{-x}{1+x}$

3. $y' = (2x+1)^5 (x^4-3)^6 \left(\frac{10}{2x+1} + \frac{24x^3}{x^4-3} \right)$

4. $\frac{1}{2}e^2 + e - \frac{1}{2}$

5. The differentiation formula is simplest.

Exponential Functions
Practice Problems

1.) Make a rough sketch of $y = 3 - e^x$

2.) Differentiate: $f(x) = x^2 e^x$

3.) Differentiate: $y = e^{ax^3}$

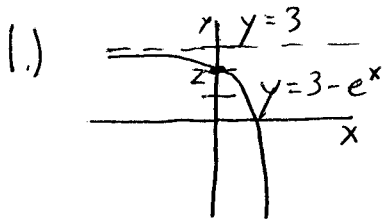
4.) Differentiate: $y = e^{t \sin t}$

5.) $\int_0^5 e^{-3x} dx = ?$

6.) $\int \frac{e^x + 1}{e^x} dx = ?$

7.) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = ?$

Solutions



$$2.) f'(x) = x(x+2)e^x$$

$$3.) y' = 3ax^2 e^{ax^3}$$

$$4.) y' = e^{t \sin 2t} (2t \cos 2t + \sin 2t)$$

$$5.) \frac{1}{3}(1 - e^{-15})$$

$$6.) x - e^{-x} + C$$

$$7.) 2e^{\sqrt{x}} + C$$

PRACTICE PROBLEMS

INVERSE FUNCTIONS

THE INVERSE OF THE FUNCTION:

1. $f(x) = 3 - 2x$

2. $f(x) = \sqrt{10 - 3x}$

3. $y = \frac{1 - \sqrt{x}}{1 + \sqrt{x}}$

FIND $(f^{-1})'(a)$

4. $f(x) = x^3 + x + 1$, $a = 1$

5. $f(x) = 3 + x^2 + \tan\left(\frac{\pi x}{2}\right)$, $-1 < x < 1$, $a = 3$

INVERSE TRIGONOMETRIC FUNCTIONS

1. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

2. $\tan^{-1}(\sqrt{3}) =$

FIND THE DERIVATIVE OF THE FUNCTION

1. $y = \tan^{-1}\sqrt{x}$

4. $y = \cos^{-1}(e^{2x})$

5. $y = \text{Arctan}(\cos \theta)$

EVALUATE THE INTEGRAL

6. $\int \frac{x+a}{x^2+a} dx$

7. $\int \frac{dx}{\sqrt{x(1+x)}}$

8. $\int \frac{t^2}{\sqrt{1-t^6}} dt$

9. $\int_0^{\sqrt{3}/4} \frac{dx}{1+16x^2}$

10. $\int_0^{1/2} \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx$

PRACTICE PROBLEM SOLUTIONS

INVERSE FUNCTIONS

1. $f^{-1}(x) = \frac{3}{2} - \frac{1}{2}x$
2. $f^{-1}(x) = -\frac{1}{3}x^2 + \frac{10}{3}, x \geq 0$
3. $y = \left(\frac{1-x}{1+x}\right)^2, -1 \leq x \leq 1$
4. 1
5. $\frac{2}{\pi}$

INVERSE TRIGONOMETRIC FUNCTIONS

1. $\frac{\pi}{3}$
2. $\frac{\pi}{3}$
3. $y' = \frac{1}{2\sqrt{x}(1+x)}$
4. $y' = -2e^{2x}/\sqrt{1-e^{4x}}$
5. $y' = \frac{-\sin \theta}{1+\cos^2 \theta}$
6. $\frac{1}{2} \ln(x^2+9) + 3 \operatorname{Arctan}\left(\frac{x}{3}\right) + C$
7. $2 \tan^{-1}(\sqrt{x}) + C$
8. $\frac{1}{3} \operatorname{Arcsin}(t^3) + C$
9. $\frac{\pi}{12}$
10. $\frac{\pi^2}{72}$