

ESP Workshop, Worksheet #1

Thursday, August 31, 2006

1. The year is 1993. Kelly is totally freaking out because she has to understand calculus during *Saved by the Bell: The College Years!* Let's help Kelly and A.C. Slater remember some of the stuff they learned at Bayside High!

- (a) Sketch the graphs of $y = \sin x$ and $y = \csc x$ on the same set of axes, and label a few points on each graph. Repeat with $y = \pi \cot x$ and $y = \pi \tan x$. Kelly says, "They look like waves!". Slater says, "They look like my muscles!" Who is right?
- (b) Give an example of a function $f : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$\lim_{x \rightarrow \infty} f(x)$$

does not exist and is not $\pm\infty$. Kelly says, "Infinity is big!" and Slater says, "Maybe, but nothing is bigger than my muscles!"

- (c) What is $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$? Can you use L'Hospital's rule here? Kelly asks, "Is that French for 'the hospital'?" Slater replies, "Yeah, that's where the guy that I L'Pinned yesterday had to L'Go."
- (d) Suppose that $\alpha > 0$. Show that $\lim_{x \rightarrow \infty} x^\alpha$ is unbounded. Slater says, "My opponents' pain is unbounded!"
- (e) Again, let $\alpha > 0$. What is

$$\lim_{x \rightarrow \infty} \frac{x^\alpha \sin x}{e^x}?$$

Kelly and Slater say, "Math is easy!" Do you agree?

2. Zack is getting jealous, so we have to teach him some calculus, too. Let $f(x) = 2x - 1$ and $g(x) = 1/x$.

- (a) Sketch the graphs of $y = f(x)$, $y = g(x)$, and $y = f(x)g(x)$ near $x = 1$. Zack says, "The graph of g looks like my gorgeous hair!"
- (b) Given your sketch from part (a), what would you guess the value of

$$\lim_{x \rightarrow 1} (fg)(x)$$

is? Can you justify your guess?

- (c) Can you use L'Hospital's rule to evaluate $\lim_{x \rightarrow 1} \frac{2x-1}{x}$?

3. Jessie, the brainiac, says, "I can't figure out the next problem and I have a test tomorrow and just in case I make the worst possible career choice ever, I need to pass!" Let's help Jessie by setting $f(x) = (\sin x)/x$, $g(x) = 1/x$, and $h(x) = f(g(x))$.

- (a) Sketch $y = h(x)$, that is, sketch $y = x \sin(1/x)$. Jessie jokes, “now that’s what I call a bad hair day!”
- (b) Are f and g continuous everywhere? Jessie reminds us that continuous basically means “not broken”.
- (c) Is h continuous everywhere? Does this contradict your answer to part (b)? Jessie wonders what can be said in general about the composition of continuous functions—what do you think?
- (d) Sketch $y = h'(x)$ without actually calculating $h'(x)$. Jessie remembers, “Oh yeah! $h'(x)$ is just the slope of the line tangent to the graph of $y = h(x)$ at the point $(x, h(x))!$ ”
- (e) Jessie has a favorite function, which she calls $J(x)$, but she won’t tell you what it is. All she’ll tell you is that $\lim_{x \rightarrow 0} J(x) = 0$. Why couldn’t you use L’Hospital’s rule to evaluate

$$\lim_{x \rightarrow 0} \frac{h(x)}{J(x)}?$$

4. Screech loves math, and he has figured out a way to send a secret calculus message from the past! Help me decode it! First, let $f(x) = \frac{1}{2}e^x$.
- (a) Sketch $y = f(x)$, $y = -f(-x)$, and $y = f(x) - f(-x)$ on the same axes. Screech says, “Sometimes, I Need Help!”
 - (b) What is $d/dx(f(x) - f(-x))$?
 - (c) Sketch $y = f(x)$, $y = f(-x)$, and $y = f(x) + f(-x)$ on the same axes. Screech says, “College is Obviously So Hard!”
 - (d) What is $d/dx(f(x) + f(-x))$?
 - (e) What do you notice about your answers for parts (b) and (d)? Does this phenomenon remind you of another famous pair of functions?
 - (f) What is Screech trying to tell you??
5. Zack and the gang are throwing a huge party tomorrow, and they need your help. The celebration is the most important party they have ever hosted. They have 1000 2-liter bottles of soda that they are planning to open for the celebration, but they know that one of them is poisoned, and they want you to find out which one it is. The poison, although harmless, exhibits no symptoms until ten to twenty hours after ingestion, at which point whoever drank it becomes “uncool”, even if that person drank just a little bit of the poisoned soda. You have lots of friends willing to try the soda, because Zack has promised that all your friends—or at least the ones who are “still cool”—can be extras on the set of *Saved by the Bell* if you find the poisoned soda. What is the smallest number of friends you must have to drink from the bottles to be absolutely sure to find the poisoned bottle within 24 hours?