ESP Workshop, Worksheet #11
Tuesday October 10, 2006
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1. Let's warm up with some lines and planes and surfaces. Let P be the plane defined by the equation $2x - y + 3z = 1$. That is, P is the set of points (x, y, z) in 3-space that satisfy the above equation. In set notation, this is:

$$P = \{(x, y, z) \in \mathbb{R}^3 : 2x - y + 3z = 1\}$$

   (a) Find an equation for the plane through the origin that does not intersect P.

   (b) Find three distinct points in P. Use these points to find a normal vector for P.

   (c) Find two other planes Q and R such that $P \perp Q$, $Q \perp R$, and $P \perp R$.

   (d) Let the line l be defined by the parametric equations:

$$x = 3 - t,\ y = 2 + t,\ z = 5t$$

   Where does l intersect P? Can you find a line that does not intersect P? What about a line that is contained in P?

   (e) Now consider the coordinate planes $x = 0$, $y = 0$, and $z = 0$. (What have we been calling these planes?) How do they intersect the surface S given by the equation $4x^2 + 9y^2 + 36z^2 = 36$? What did Professor Durbin call these intersections in class yesterday? Can you use them to guess what S looks like in three dimensions?

2. Here are eight equations and eight pictures. Match the equations with the surfaces they define.

   (a) $y^2 - z^2 = 1 + x^2$ (b) $x^2 = 1 - 2z^2$ (c) $2x^2 - 2y^2 + 2z^2 = 2$ (d) $9x^2 + 4y^2 + z^2 = 1$

   (e) $z^2 = y - 2x^2$ (f) $x^2/36 + y^2/9 + z^2/4 = 1/36$ (g) $y + z^2 = x^2$ (h) $0 = x^2 + 2z^2 - y^2$

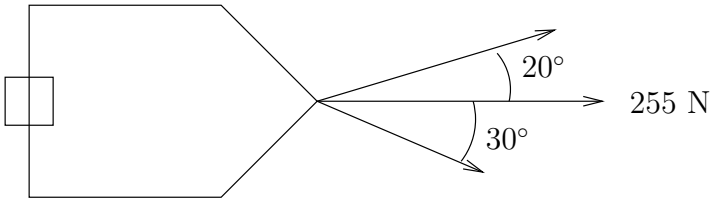

Figure 1: This is a picture of a boat for question 3.

3. A boat is pulled onto shore using two ropes, as shown in Figure 2. If a force of 255 N is needed, find the magnitude of force in each rope.

4. Yesterday, I was telling a friend about Colt McCoy's amazing 3rd quarter against Oklahoma, but my friend had forgotten what a "football" was. However, he's good at math, so all I had to say was "$x^2 + 4y^2 + 4z^2 = 1$" and then he remembered what sport I was talking about. (Keep in mind: he's good with both equations and inequalities.) How could you tell this guy about...

   (a) ...soccer?

   (b) ...hockey?

   (c) ...rugby?

   (d) ...frisbee?

   (e) ...horseback riding? (Describe the saddle.)

   (f) ...snowboarding in a half-pipe?

   (g) ...skateboarding in an empty pool?

   (h) (Challenge.) ...golf? (Yes, dimples and everything.)

5. Each edge of a cubical box has length 1 m. The box contains nine spherical balls—eight orange and one white—all with the same radius r. The center of the white ball is at the center of the cube and it touches the other eight balls, each of which is near one of the eight corners of the box. Each of the orange balls touches three sides of the box. Thus, the balls are tightly packed in the box. Find r.

6. A plane is capable of flying at a speed of 180 km/h in still air. The pilot takes off from an airfield and heads due north according to the plane's compass. After 30 minutes of flight time, the pilot notices that, due to the wind, the plane has actually traveled 80 km at an angle 5° east of north.

   (a) What is the wind velocity?

   (b) In what direction should the pilot have headed to reach the intended destination?