ESP Workshop, Worksheet #12

Thursday October 12, 2006

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1. Hey! You've got an exam coming up! Let's practice stuff we know with some of Durbin's old exam problems!

   (a) Find the radius of convergence and the interval of convergence for

   $$\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n3^n}$$

   (b) Find the Taylor series centered at $c = \pi/2$ for $f(x) = \sin x$. Express your answer using $\sum$ notation.

   (c) Using polar coordinates, draw the graphs of $r = 2$ and $r = 2 + \sin\theta$.

   (d) Let $\mathbf{u} = \langle -5, 12 \rangle$ and $\mathbf{v} = \langle 3, 4 \rangle$. Find: (i) the angle between $\mathbf{u}$ and $\mathbf{v}$, (ii) the projection of $\mathbf{u}$ onto $\mathbf{v}$, and (iii) the vector component of $\mathbf{u}$ orthogonal to $\mathbf{v}$.

2. We just learned about cylindrical and spherical coordinates in three-space. As a warm-up, let's compare some graphs in rectangular and polar coordinates in two-dimensional space. Graph each equation on its own set of axes.

   (a) In rectangular coordinates (x, y) : $x = 5$; $x^2 + 4y^2 = 1$

   (b) In polar coordinates (r, θ) : $r = 5$; $\theta = \pi/4$; $r = 2 + \sin(4\theta)$

   (c) Recall that we have the following relationships between polar (r, θ) and rectangular coordinates (x, y):

   $$r^2 = x^2 + y^2, \ \tan\theta = \frac{y}{x}$$

   $$x = r\cos\theta, \ y = r\sin\theta$$

   Can you write the equation $r = 2 + \sin(4\theta)$ in rectangular coordinates? (Hint: it ain't pretty.)

3. Let's practice using the three coordinate systems we know for three-dimensional space. For each of the following, describe the surface and draw it (if you can).

   (a) In rectangular coordinates (x, y, z) : $x = 5$; $4x^2 + 4y^2 + z^2 = 1$

   (b) In cylindrical coordinates (r, θ, z) : $r = 2$; $r = 2 + \sin\theta$; $\theta = \pi/2$; $4r^2 + z^2 = 1$

   (c) In spherical coordinates (ρ, θ, ϕ) : $\rho = 2$; $\rho = 2 + \sin(4\theta)$; $\rho = 2 + \sin(4\phi)$

   (d) Can you describe a golf ball using spherical coordinates?

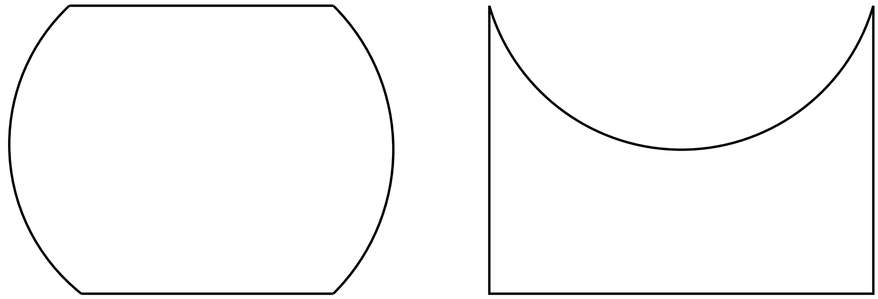

Figure 1: One the left is a cross-section of a balance beam, and on the right is one of the chalk holder.

(e) (A Geography/Zoology question!) A bear is sitting at some point N on the Earth. He gets up and walks one mile south, then one mile east, and finally one mile north. If the bears ends up at N, the same point where he started, then what color is the bear?

4. It's solid-sketching time! Also, describe what each solid could be used for.

   (a) In rectangular coordinates $(x, y, z) : 1 \leq |x| \leq 2;\ 1 \leq |y| \leq 2; 1 \leq |z| \leq 2$

   (b) In cylindrical coordinates $(r, \theta, z) : 1 \leq r \leq 2;\ 0 \leq \theta \leq \pi;\ 0 \leq z \leq r - 1$

   (c) In spherical coordinates $(\rho, \theta, \phi) : 1 \leq \rho \leq 2;\ 0 \leq \theta \leq \pi;\ \pi/2 \leq \phi \leq \pi(r-1)/6 + \pi/2$

   (d) How are (b) and (c) similar? How are they different?

5. Kelly, one of the ESP calculus leaders from last year, is a gymnast. Use inequalities in the appropriate coordinate system to describe...

   (a) ...a tumbling mat (i.e. a very short box).

   (b) ...the uneven bars.

   (c) ...the balance beam (assume that the cross-section looks like that in Figure 1.

   (d) ...the rings.

   (e) ...the chalk-holder (looks like a steel drum—see the cross-section in Figure 1).

6. Each edge of a cubical box has length 1 m. The box contains nine spherical balls—eight orange and one white—all with the same radius r. The center of the white ball is at the center of the cube and it touches the other eight balls, each of which is near one of the eight corners of the box. Each of the orange balls touches three sides of the box. Thus, the balls are tightly packed in the box. Find r.