

ESP Workshop, Worksheet #13
Tuesday October 17, 2006
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1. Here are some practice exam-ish problems... it would be a good idea to practice writing clean, logical solutions, complete with proper symbols. Ask Eric if you have any questions about this!

- (a) Find the radius of convergence and interval of convergence of the series:

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n2^n}$$

- (b) Find the Taylor series for $f(x)$ centered at the given value of a .

$$f(x) = \ln x, \quad a = 2$$

- (c) Use the binomial series to expand the function as a power series. State the radius of convergence.

$$\sqrt[4]{1-8x}$$

- (d) Show that the equation represents a sphere, and find its center and radius.

$$x^2 + y^2 + z^2 = x + y + z$$

- (e) Let $\mathbf{u} = \langle 2, 3 \rangle$ and $\mathbf{v} = \langle 0, 5 \rangle$. Compute $\mathbf{u} + 2\mathbf{v}$, $\mathbf{u} \cdot \mathbf{v}$, and $\text{proj}_{\mathbf{v}} \mathbf{u}$ and draw pictures for the first and last ones. Why doesn't it make much sense to draw a picture about the dot product?

- (f) Let $\mathbf{a} = \langle 1, 2, 3 \rangle$ and $\mathbf{b} = \langle -1, 1, -1 \rangle$. Find the angle between \mathbf{a} and \mathbf{b} by computing $\mathbf{a} \cdot \mathbf{b}$ and also by computing $\mathbf{a} \times \mathbf{b}$.

- (g) Find an equation for the plane that passes through the point $(6, 0, -2)$ and contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$.

- (h) Plot the point whose cylindrical coordinates are $(4, -\pi/3, 5)$. Then find the rectangular coordinates of the point.

2. (Fibonacci numbers!) Show that the Maclaurin series of the function

$$f(x) = \frac{x}{1-x-x^2} \quad \text{is} \quad \sum_{n=1}^{\infty} f_n x^n$$

where f_n is the n th Fibonacci number, i.e. $f_1 = 1$, $f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$. [Hint: Write $x/(1-x-x^2) = c_0 + c_1x + c_2x^2 + \dots$ and multiply both sides of this equation by $1-x-x^2$.]

Now obtain the Maclaurin series for $f(x)$ in a different way by writing $f(x)$ as a sum of partial fractions. Use this and your previous answer to find an explicit formula for f_n .

3. Sketch the following surfaces:

$$x^2 + 9z^2 = 36$$

$$y^2 + 4z^2 = 1 + x^2$$

$$y^2 = 1 + 4x^2 + 9z^2$$

$$x + y^2 = z^2$$

$$x^2 = y^2 + z^2$$