ESP Workshop, Worksheet #14

Tuesday October 24, 2006

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1. For each of the vector-valued functions given below, find the limit. Remember, a vector-valued function is of the form

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

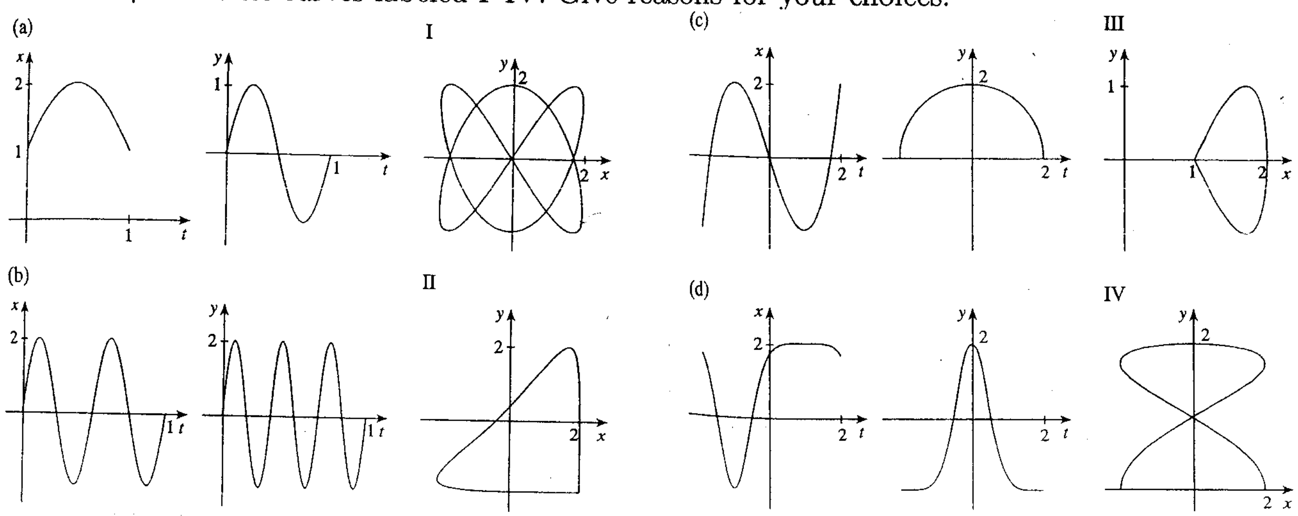
where f, g, and h are real-valued functions, i.e. $f, g, h : \mathbb{R} \to \mathbb{R}$.

   (a) $\lim_{t\to 0^+} \langle \cos t, \sin t, t \ln t \rangle$

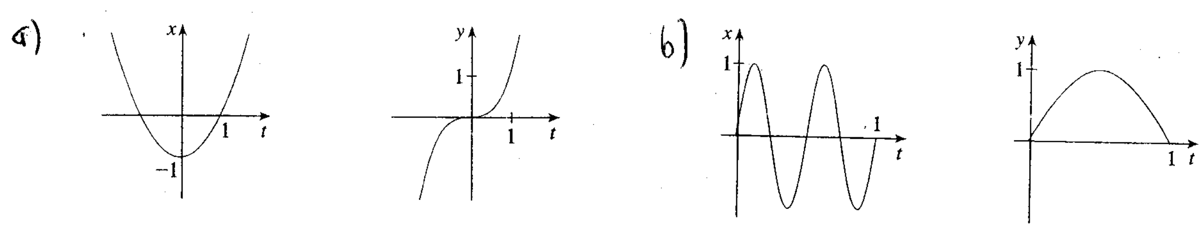
   (b) $\lim_{t\to 0} \langle \frac{e^t - 1}{t}, \frac{\sqrt{1+t}-1}{t}, \frac{3}{1+t} \rangle$

   (c) $\lim_{t\to 1} \left(\sqrt{t+3}\mathbf{i} + \frac{t-1}{t^2-1}\mathbf{j} + \frac{\tan t}{t}\mathbf{k}\right)$

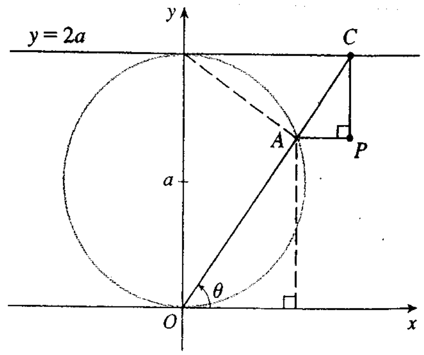
2. Match the graphs of the parametric equations $x = f(t)$ and $y = g(t)$ in (a)–(d) with the parametric curves labeled I–IV. Give reasons for your choices.



3. Use the graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate with arrows the direction in which the curve is traced as t increases.



4. A curve, called a **witch of Maria Agnesi**, consists of all possible positions of the point P in the figure. (See also the movie...)



(a) Show that parametric equations for this curve can be written as

$$x = 2a\cot\theta \quad y = 2a\sin^2\theta$$

(b) Sketch the curve.

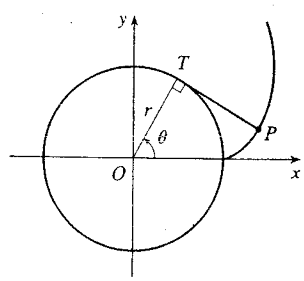
(c) Recall that Simpson's Rule says that

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

where n is even and $\Delta x = (b-a)/n$. Use Simpson's Rule with $n = 4$ to estimate the length of the arc of this curve with $\pi/4 \leq \theta \leq \pi/2$.

5. A string is wound around a circle and then unwound while being held taut. The curve traced by the point P at the end of the string is called the **involute** of the circle. If the circe has radius r and center O and the initial position of P is $(r, 0)$, and if the parameter θ is chosen as in the figure, show that the parametric equations of the involute are

$$x = r(\cos\theta + \theta\sin\theta) \quad y = r(\sin\theta - \theta\cos\theta)$$



6. (An Agricultural question!) A cow is tied to a silo with radius r by a rope just long enough to reach the opposite side of the silo. First draw a picture of the situation, and then find the area available for grazing by the cow. (Hint: use the ideas from the previous question...)