ESP Workshop, Worksheet #16
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AI: Eric Katerman

OMG! In fact, ZOMG!!! You've got *another* exam coming up! Don't be a N00b! In order to avoid getting PWNED by the l33t math wizard Doctor Durbin, let's practice with some of his old exam problems... w00t!

1. Consider the curve represented parametrically by the equations

$$x = t^2 - 4t, \qquad y = t^2$$

   (a) Sketch the graph for $0 \leq t \leq 2$. (Suggestion: use the graph paper provided.)

   (b) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

   (c) Find all points (if there are any) of horizontal and vertical tangency. [Do not restrict t as in part (a).]

2. (a) Draw the graph of $r = 2 + \sin\theta$.

   (b) Write but do not evaluate an integral that will give the length of the graph in part (a) over $0 \leq \theta \leq 2\pi$.

   (c) Write but do not evaluate an integral that will give the area inside the graph in part (a).

3. The acceleration of an object at each time t is given by $\mathbf{a}(t) = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Also, $\mathbf{v}(0) = 3\mathbf{k}$ and $\mathbf{r}(0) = \mathbf{j}$ ($\mathbf{v}$ for velocity, $\mathbf{r}$ for position). Find $\mathbf{r}(t)$ and the position at time $t = 2$.

4. Find a set of parametric equations for the line tangent to the curve $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j} + \mathbf{k}$ at (4,2,1).

5. Sketch a contour map for $f(x, y) = 2x^2 + y$ using the level curves corresponding to $c = 0, -1$, and 2.

6. Use the limit definition of partial derivative to find $f_x(x, y)$ given $f(x, y) = x^2y^3$. (Write your solution very carefully. For example, do not be sloppy with limit notation.)

7. Given $f(x, y) = 2xy^3$, find $f_y(x, y)$ by forming the appropriate difference quotient and taking the limit as h tends to zero.

8. Find the equation of the tangent plane to the surface $x^2 - y^2 - z^2 = 4$ at (3,1,-2).

9. (A Chemistry problem!) The gas law for a fixed mass m of an ideal gas at absolute temperature T, pressure P, and volume V is $PV = mRT$, where R is the gas constant.

   (a) Show that
   $$\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1$$

   (b) For the ideal gas of part (a), show that
   $$T\frac{\partial P}{\partial T}\frac{\partial V}{\partial T} = mR$$

   (c) Now assume that the pressure, volume, and temperature of a mole of an ideal gas are related by the equation $PV = 8.31T$, where P is measured in kilopascals, V in liters, and T in kelvins. Use differentials to find the approximate change in the pressure if the volume increases from 12 L to 12.3 L and the temperature decreases from 310 K to 305 K.

10. (An Electrical Engineering problem!) If R is the total resistance of the three resistors, connected in parallel, with resistances R_1, R_2, R_3, then
   $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

   (a) Rewrite this equation to make R a function of R_1, R_2, and R_3. (All I mean is that you should invert both sides and possibly simplify to get $R = f(R_1, R_2, R_3)$.)

   (b) Use your solution to part (a) to find $\partial R/\partial R_1$, i.e. the partial derivative of the function R with respect to the variable R_1.

   (c) If the resistances are measured in ohms as $R_1 = 25\ \Omega$, $R_2 = 40\ \Omega$, and $R_3 = 50\ \Omega$, with a possible error of 0.5% in each case, estimate the maximum error in the calculated value of R.

11. (A Marine Biology problem!) Marine biologists have determined that when a shark detects the presence of blood in the water, it will swim in the direction in which the concentration of blood (in parts per million) at a point $P(x, y)$ on the surface of the seawater is approximated by
   $$C(x, y) = e^{-(x^2+2y^2)/10^4}$$
   where x and y are measured in meters in a rectangular coordinate system with the blood source at the origin.

   (a) Identify the level curves of the concentration function and sketch several members of this family together with a path that the shark will follow to the source.

   (b) (Challenge.) Suppose a shark is at the point (x_0, y_0) when it first detects the presence of blood in the water. Find and equation of the shark's path by setting up and solving a differential equation.