ESP Workshop, Worksheet #18
Thursday November 9, 2006
AI: Eric Katerman

As promised, here are some more old exam problems...

1. Use the Chain Rule to find $\partial u/\partial s$ if
$$u = x^2 - y^2,\ x = s\cos t,\ \text{and } y = t\cos s$$
Write the answer in terms of s and t.

2. The length x and width y of a rectangular parallelopiped are increasing at a rate of 3 cm per second, and the height z is increasing at a rate of 2 centimeters per second. At what rate is the volume changing when the length, width, and height are 20, 15, and 10 cm respectively?

3. Find and simplify $\partial z/\partial y$ if $xe^z - ze^y = 0$.

4. The radius r and height h of a right circular cylinder are changing in such a way that the volume V is increasing at a rate of 2 cubic centimeters per second when the height is 12 centimeters and the radius is 3 centimeters. Find the rate of change of the height if the radius is increasing at a rate of 2 centimeters per second. ($V = \pi r^2 h$)

5. Find the directional derivative of $f(x, y) = xy^2$ at (2,-1) in the direction $\mathbf{v} = \mathbf{i} - \mathbf{j}$.

6. Find the equation of the tangent plane to the surface $x^2 - y^2 - z^2 = 4$ at the point (3,1,-2).

7. Find the path of a heat-seeking particle placed at the point (-2,3) on a metal plate with the temperature field $T(x, y) = 20 - x^2 - 4y^2$.

8. Assume $f(x, y) = xe^y$.
   (a) Find the gradient of f.
   (b) Find the directional derivative of f at (-2,1) in the direction $3\mathbf{i} + 4\mathbf{j}$.
   (c) What is the maximum value of the directional derivative of f at (-2,1)?

9. (Challenge!) Among all planes that are tangent to the surface $xy^2z^2 = 1$, find the ones that are farthest from the origin.

10. (Challenge!) Suppose that f is a differentiable function of one variable. Show that all tangent planes to the surface $z = xf(y/x)$ intersect in a common point.