ESP Workshop, Worksheet #7
Thursday September 21, 2006
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1. Warm-up: Screech loves math, and he has figured out a way to send a secret calculus message from the past! Help me decode it! First, let $f(x) = \frac{1}{2}e^x$.

   (a) Sketch $y = f(x), y = -f(-x)$, and $y = f(x) - f(-x)$ on the same axes. Screech says, "Sometimes, I Need Help!"

   (b) What is $d/dx(f(x) - f(-x))$?

   (c) Sketch $y = f(x), y = f(-x),$, and $y = f(x) + f(-x)$ on the same axes. Screech says, "College is Obviously So Hard!"

   (d) What is $d/dx(f(x) + f(-x))$?

   (e) What do you notice about your answers for parts (b) and (d)? Does this phenomenon remind you of another famous pair of functions?

   (f) What is Screech trying to tell you??

2. Suppose we have a series $\sum_{n=1}^{\infty} a_n$. What does it mean for that sum to be absolutely convergent? What about conditionally convergent? Give an example of an alternating series that is absolutely convergent and one that is conditionally (but not absolutely) convergent. Can you think of an example that absolutely converges but does not converge??

Okay, enough Mr. Nice Guy. Let's see what you're really made of...

3. Find the limit of the sequences:

$$(a)\left\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots\right\} \qquad (b)\left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$$

4. Find the values of p for which the series is convergent.

$$(a)\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^p} \qquad (b)\sum_{n=3}^{\infty}\frac{1}{n\ln n[\ln(\ln n)]^p} \qquad (c)\sum_{n=2}^{\infty}\frac{1}{n^p\ln n}$$

5. Find all positive values of x such that the series

$$\sum_{n=1}^{\infty} x^{\ln n}$$

converges. Is this a power series?

6. For what values of p is each series convergent?

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p} \qquad (b) \sum_{n=1}^{\infty} \frac{(-1)^n}{n+p}$$

7. For which positive integers k is the following series convergent?

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$$

8. (Challenge.) Find the sum of the series

$$\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$$

(Super challenge.) Can you tell me how this method of problem-solving relates to the famous Riemann-zeta function (shown below)?

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

Do you know the value (possibly ∞, hint hint) of this function for any s? Later this semester, we will compute $\zeta(2)$. Do you happen to know what it is? If so, can you derive it?? By the way, if you understand this function really well, you could win a million dollars!! (See Eric for details.)